NTNU – Trondheim
Norwegian University of Science and Technology

Stochastic Volatility Models Predictive Relevance for Equity Markets

Predicting step ahead Equity Market Volatility

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Professor
Dr.oecon Per B Solibakke

SV models predictive relevance

Objectives

- A stochastic volatility model should be able to predict market volatility
- The use of such volatility models entail forecasting aspects of future returns
    - magnitude of returns,
    - predict quantiles and
    - entire density
- Volatility measures are important factors for:
    - Equity markets, Currency markets, Commodity markets
    - Several other aspects in Corporate and Managerial finance

SV models predictive relevance

Asset / Index Volatility relevance for Equity markets

- Risk Management (market, credit and operational risk)
- Portfolio selection and CAPM
- Derivatives (hedging and speculation)
- Market making (i.e. bid-ask spread)
- Market timing (i.e buying or selling a tracking portfolio)
- Several objectives in corporate / managerial finance
  - Event studies
  - Mergers & Acquisitions
  - Management Accounting and Pricing

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- Volatility is not observable (latent)
- Stochastic Volatility models explicitly describe multifactor volatility showing well-known characteristics:
  - Extreme tails (the power law)
  - Volatility clustering
  - Volatility persistence
  - Mean reversion
  - Volatility asymmetry
  - Long memory

- Any evaluation of forecasting performance will be difficult. However, well-known standard models are available.

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Stochastic Volatility models:

Starting point:

$$\frac{dS_t}{S_t} = \left(\mu + c \cdot \left(V_{1,t} + V_{2,t}\right)\right) dt + \sqrt{V_{1,t}}\, dW_{1,t} + \sqrt{V_{2,t}}\, dW_{2,t}$$

where $V_{i,t}, i = 1, 2$ is either log linear or square root (affine) functions, $dW_{i,t}, i = 1, 2$ is normally distributed variables which can be correlated

$$\rho = corr\left(dW_{1,t}; dW_{2,t}\right)$$

SV models predictive relevance

The Stochastic Volatility model (and implemented in C/C++):

$$y_t = a_0 + a_1\left(y_{t-1} - a_0\right) + \exp(V_{1t} + V_{2t}) \cdot u_{1t}$$
$$V_{1t} = b_0 + b_1\left(V_{1,t-1} - b_0\right) + u_{2t}$$
$$V_{2t} = c_0 + c_1\left(V_{2,t-1} - c_0\right) + u_{3t}$$
$$u_{1t} = dW_{1t}$$
$$u_{2t} = s_1\left(r_1 \cdot dW_{1t} + \sqrt{1 - r_1^2} \cdot dW_{2t}\right)$$
$$u_{3t} = s_2\left(\begin{array}{l} r_2 \cdot dW_{1t} + \left((r_3 - (r_2 \cdot r_1)) / \sqrt{1 - r_1^2}\right) \cdot dW_{2t} + \\ \sqrt{1 - r_2^2 - \left((r_3 - (r_2 \cdot r_1)) / \sqrt{1 - r_1^2}\right)^2} \cdot dW_{3t} \end{array}\right)$$

The correlation specifications (u_{it}, i=1,2,3) use the Cholesky decomposition for model consistency and ease of interpretation.

SV models predictive relevance

FTSE100 Index / EQUINOR Asset Scientific Equity SV Model

Parameter values and Scientific Mode.

Φ	Mode	Mean	Standard error
a_0	0.00000	0.00000	0.00000
a_1	0.00000	0.00000	0.00000
b_0	0.00000	0.00000	0.00000
b_1	0.00000	0.00000	0.00000
c_1	0.00000	0.00000	0.00000
s_1	0.00000	0.00000	0.00000
s_2	0.00000	0.00000	0.00000
r1	0.00000	0.00000	0.00000
r_2	0.00000	0.00000	0.00000
r_3	0.00000	0.00000	0.00000
Distributed Chi-square (number of freedoms)			χ^2(?)
Posterior at the mode			-yyyyy
Chi-square test statistic			{xxxxx}

+ rejection rates, parameter chains and log posterior chains …..

SV models predictive relevance

Post-Optimal SV model Analysis:

We proceed backwards to infer the unobserved state vector from the observed process as implied by our particular SV- model.

➔ The non-linear Kalman filter.

SV models predictive relevance

The non-linear Kalman filter methodology:

1. The Metropolis-Hastings algorithm and parallel computing has been employed for the estimation of the stochastic volatility model and the parameter estimate.
   $$\phi = (a_0, a_1, b_0, b_1, s_1, c_0, c_1, s_2, r_1, r_2, r_3)$$
   A by-product is a 250 *k* simulation realisation.

2. From this 250 *k* simulation, a reduced form auxiliary model is established with a tractable likelihood function. The *SNP* provides a convenient representation of the one-step ahead conditional variance.

3. Finally, we move backwards to infer the unobserved state vector from the observed process as implied by our model.

SV models predictive relevance

The Theory and the nonlinear Kalman filter methodology:

1. Elicit the dynamics of the implied conditional density for observables (SNP round 1):

$$\hat{p}\left(y_0 \mid x_{-1}\right) = p\left(y_0 \mid x_{-1}, \hat{\phi}_n\right)$$

where x_{-1} is the lagged state vector. An unconditional expectation is

$$E_{\hat{\phi}_n}\left(g\right) = \int \ldots \int g\left(y_{-L}, \ldots, y_0\right) p\left(y_{-L}, \ldots, y_0 \mid \hat{\phi}_n\right) dy_{-L}, \ldots dy_0$$

and can be computed by generating a simulation $\left\{\hat{y}_t\right\}_{t=-L}^{N}$, from system with parameters $\hat{\phi}_n$ and using .

$$E_{\hat{\phi}_n}\left(g\right) = \frac{1}{N} \sum_{t=0}^{N} g\left(\hat{y}_{-L}, \ldots, \hat{y}_t\right)$$

SV models predictive relevance

The Theory and the nonlinear Kalman filter methodology:

2. With respect to the unconditional expectation so computed, define

$$\hat{\theta}_k = \underset{\theta \in \Re^{pK}}{argmax} \; E_{\hat{\phi}_n} \log f_K\left(y_0 \mid x_{-1}, \theta\right)$$

where $f_K\left(y_0 \mid x_{-1}, \theta\right)$ is the *SNP* density (round 2). Let

$$\hat{f}_K\left(y_0 \mid x_{-1}\right) = f_K\left(y_0 \mid x_{-1}, \hat{\theta}_K\right)$$

Gallant and Long (1997) theorem 1 states:

$$\lim_{K \to \infty} \hat{f}_K\left(y_0 \mid x_{-1}\right) = \hat{p}\left(y_0 \mid x_{-1}\right)$$

Convergence is with respect to a weighted Sobolev norm.

SV models predictive relevance

Implementation procedure (part 1):

From the SV model estimation, as a by-product, a long simulated realization of the state vector $\left\{\hat{V}_{i,t}\right\}_{t=1}^{N}, i = 1, 2$ and the corresponding $\left\{\tilde{y}_t\right\}_{t=1}^{N}$ for $\phi = \hat{\phi}$, are available.

Hence, the *N* = 250 *k* SV model simulations, $\left\{\hat{y}_\tau\right\}_{\tau=1}^{250k}$ calibrate the functional form of the conditional distribution of functions of $\hat{V}_t$.

SV models predictive relevance

Implementation procedure (part 2):

Estimate an *SNP* model on the $\hat{y}_t$ because the model provides a convenient representation of the one-step ahead conditional variance $\hat{\sigma}_t^2$ of $\hat{y}_{t+1}$ given $\{\tilde{y}_\tau\}_{\tau=1}^{t}$, where *t* = 250 *k*.

Run regressions of $\hat{V}_{i,t}, i=1,2$ on $\hat{\sigma}_t^2$, $\hat{y}_t$ and $|\hat{y}_t|$ and generously long lags of these series, establishing general functions from the simulated SV model values.

For step ahead of $V_{1,t}$ *and* $V_{2,t}$ it is allowed to use very general functions

of $\{y_\tau\}_{\tau=1}^{250k}$ and obviously, there a huge data set available.

SV models predictive relevance

Implementation procedure (part 3):

Finally, these functions are evaluated on the observed data series $\left\{\tilde{y}_\tau\right\}_{\tau=1}^{t}$ giving predicted values for $\tilde{V}_{1,t}$ *and* $\tilde{V}_{2,t}$ for the two volatility factors at the data points.

The yearly volatility is $\exp\left(\tilde{V}_{1,t} + \tilde{V}_{2,t}\right)$ multiplied by the square root of the number of trading days in a year (i.e. 252 trading days).

From previous research for equity markets:

$\tilde{V}_{1,t}$ is a slowly moving process (extremely persistent)

$\tilde{V}_{2,t}$ is a choppy process (strongly mean reverting)

SV models predictive relevance

Applications Equity Markets:

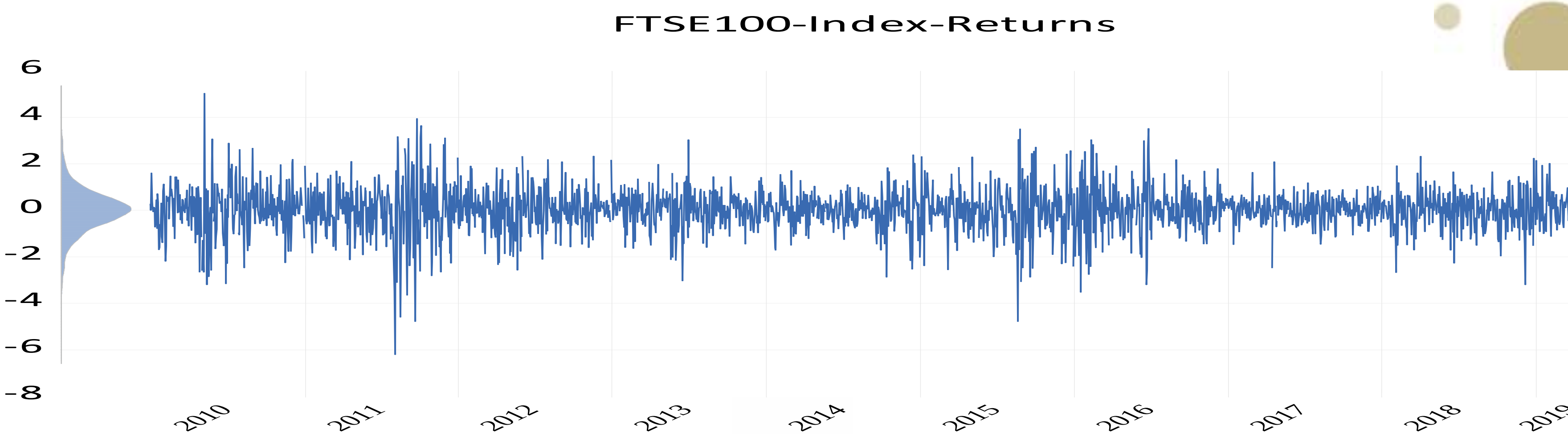
Two European equity market series to exemplify the (latent) volatility predictions:

1. FTSE100 Index (London)

2. The Equinor Share Asset (Oslo)

The data series examples indicate that the methodology can be applied to equities, currencies, cryptos and commodities (spot series as well as forwards/futures). Moreover, in this presentation I use daily closing prices. Other price intervals is applicable (i.e. 5 minutes, one hour, one day, one week).

SV models predictive relevance



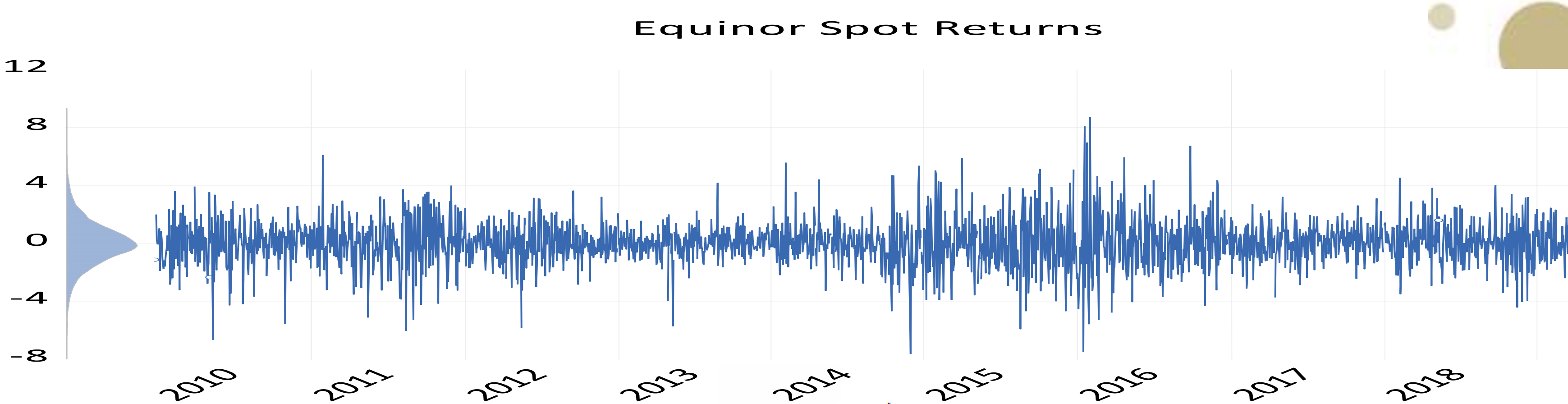
Included observations: 2342

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat
		1	0.027	0.027	1.7560
		2	-0.044	-0.045	6.3876
		3	0.022	0.025	7.5706
		4	-0.074	-0.077	20.279
		5	-0.028	-0.021	22.060
		6	0.007	0.001	22.175
		7	-0.016	-0.015	22.751
		8	-0.040	-0.043	26.421
		9	0.009	0.006	26.606
		10	-0.002	-0.006	26.618
		11	-0.011	-0.011	26.913
		12	-0.003	-0.010	26.936
		13	-0.007	-0.009	27.068
		14	-0.006	-0.006	27.159
		15	-0.018	-0.021	27.905
		16	0.011	0.009	28.170
		17	-0.014	-0.017	28.617
		18	-0.022	-0.021	29.716
		19	0.007	0.002	29.842
		20	0.008	0.006	30.001

Included observations: 2342

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat
		1	0.219	0.219	112.49
		2	0.191	0.151	198.32
		3	0.219	0.161	310.49
		4	0.208	0.126	411.73
		5	0.189	0.092	495.87
		6	0.133	0.021	537.48
		7	0.133	0.030	578.99
		8	0.139	0.041	624.48
		9	0.169	0.081	691.88
		10	0.103	-0.000	716.94
		11	0.145	0.060	766.53
		12	0.083	-0.025	782.60
		13	0.132	0.049	823.39
		14	0.066	-0.034	833.67
		15	0.058	-0.017	841.60
		16	0.119	0.056	875.17
		17	0.130	0.067	914.88
		18	0.070	-0.013	926.53
		19	0.113	0.051	956.90
		20	0.134	0.050	999.61

SV models predictive relevance



Included observations: 2328

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat
		1	-0.019	-0.019	0.8144
		2	-0.062	-0.062	9.6910
		3	-0.022	-0.024	10.818
		4	-0.017	-0.022	11.522
		5	0.007	0.003	11.623
		6	-0.016	-0.019	12.256
		7	-0.029	-0.030	14.210
		8	0.011	0.007	14.498
		9	0.005	0.001	14.567
		10	0.022	0.021	15.707
		11	0.040	0.041	19.510
		12	-0.014	-0.009	19.959
		13	-0.047	-0.043	25.212
		14	-0.028	-0.030	27.053
		15	0.018	0.013	27.818
		16	0.028	0.023	29.601
		17	-0.004	-0.001	29.637
		18	-0.045	-0.041	34.358
		19	0.015	0.011	34.869
		20	0.027	0.019	36.539

Included observations: 2328

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat
		1	0.165	0.165	63.661
		2	0.174	0.151	134.03
		3	0.101	0.054	157.85
		4	0.130	0.088	197.39
		5	0.127	0.080	235.11
		6	0.126	0.069	272.34
		7	0.115	0.054	303.33
		8	0.126	0.065	340.61
		9	0.196	0.137	430.86
		10	0.068	-0.024	441.72
		11	0.109	0.032	469.47
		12	0.086	0.023	486.93
		13	0.097	0.022	509.01
		14	0.115	0.046	539.81
		15	0.069	-0.010	550.94
		16	0.118	0.054	583.40
		17	0.089	0.016	602.00
		18	0.075	-0.010	615.31
		19	0.083	0.030	631.62
		20	0.100	0.033	655.11

SV models predictive relevance

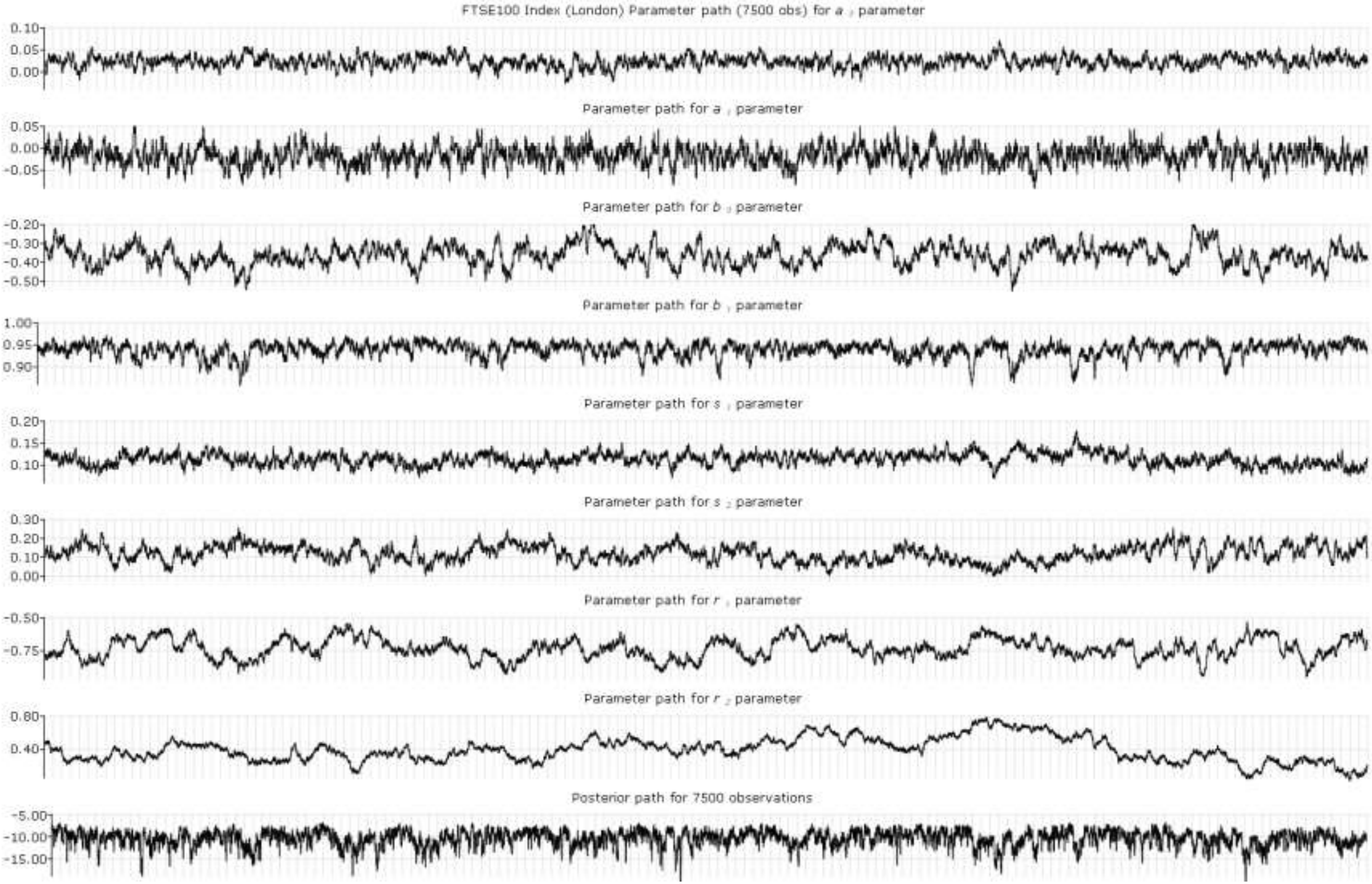
The MCMC chains for $\phi = (a_0, a_1, b_0, b_1, s_1, c_0, c_1, s_2, r_1, r_2, r_3)$ of the FTSE100 index and the Equinor asset price is reported next.

The parameter chains are choppy and the rejection rates seem appropriate (tuning the parameter increments).

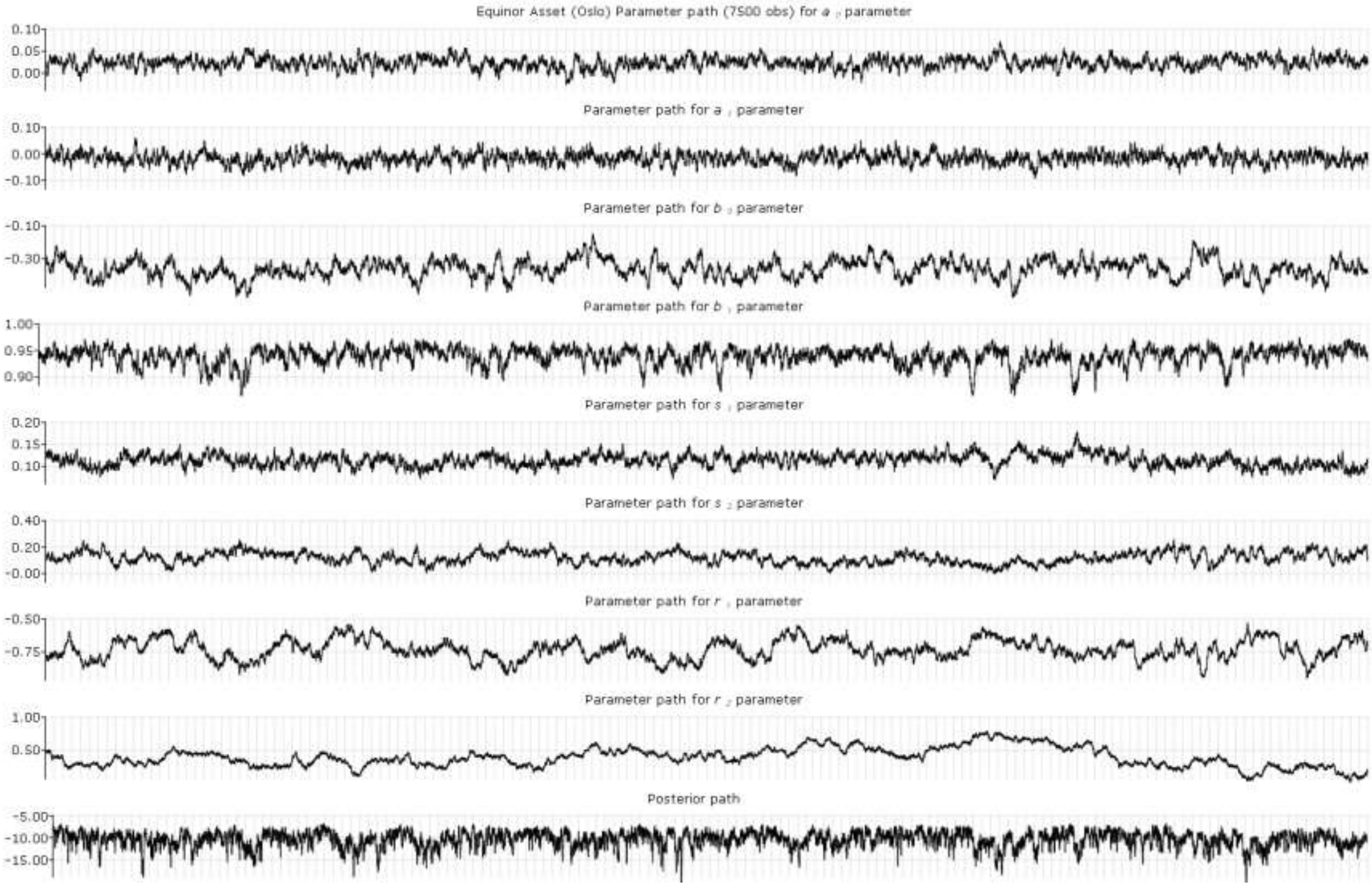
The log posterior at the mode is -6.8302 and -3.2559 for the FTSE100 index and the Equinor asset, respectively. The Chi-square test statistic with the degrees of freedom equal the length of θ minus the length of ϕ minus one, that is, 4 and 3 and therefore the *p*-values are about 0.1451 and 0.3538, respectively (do not fail the test of over identified restrictions at the level of 10%).

The SV model representation of the FTSE100 Index and the Equinor Asset seems therefore quite satisfactory.

SV models predictive relevance



SV models predictive relevance



SV models predictive relevance

The next step is to proceed backward to infer the observed state vector from the observed process as implied by the model represented with the parameter vectors below:

$$\hat{\phi} = \left(a_0, a_1, b_0, b_1, c_0, c_1, s_1, s_2, r_1, r_2, r_3\right)$$

FTSE100 Spot Index Scientific Model
Parameter values and Scientific Mode.

ϕ	Mode	Mean	Standard error
a_0	0.0190820	0.0230030	0.0127040
a_1	-0.0095730	-0.0133460	0.0214790
b_0	-0.3269400	-0.3553700	0.0633250
b_1	0.9446700	0.9357600	0.0228010
c_1	0	5E-31	0
s_1	0.1182100	0.1151500	0.0147310
s_2	0.1102300	0.1384700	0.0431090
r_1	-0.7839800	-0.7699300	0.0666170
r_2	0.48764	0.36971	0.11124

Distributed Chi-square (no.of freedom)	$\chi^2(4)$
Posterior at the mode	-6.83020
Chi-square test statistic	{0.14514}

Equinor Asset Spot Price Scientific Model
Parameter values and Scientific Mode.

ϕ	Mode	Mean	Standard error
a_0	-0.0078125	-0.0183630	0.0370070
a_1	-0.0546880	-0.0534440	0.0230030
b_0	0.4531200	0.3523000	0.1624800
b_1	0.8281200	0.8048300	0.0788550
c_1	0.0000000	0.0000000	0.0000000
s_1	0.1796900	0.1778900	0.0379900
s_2	0.1328100	0.1153900	0.0654660
r_1	-0.3437500	-0.2545200	0.2047600
r_2	0.4687500	0.4220500	0.3361000

Distributed Chi-square (no. of freedom)	$\chi^2(3)$
Posterior at the mode	-3.25590
Chi-square test statistic	{0.35383}

SV models predictive relevance

The nonlinear Kalman filter:

Step 1: Generate a new *SNP* model for $\hat{y}_t$ (SV-model simulated data) and obtain $\hat{\sigma}_t^2$ of $\hat{y}_{t+1}$ given $\{\tilde{y}_\tau\}_{\tau=1}^{250.k}$.

The representation becomes (250 *k*): ($\hat{\sigma}_t^2$ of $\hat{y}_{t+1}$)

yt	V1t	V2t	Var(yt)
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

FTSE100 Index / Equinor Asset

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

SV models predictive relevance

The nonlinear Kalman filter:

Step 2: Run ordinary regressions of $\hat{V}_{i,t}, i = 1, 2$ on $\hat{\sigma}_t^2$, $\hat{y}_t$ and $\left|\hat{y}_t\right|$ and generously long lags of these series.

Hence, inside the simulation, functions are calibrated giving predicted values for $\hat{V}_{i,t}, i = 1, 2$ given $\left\{\tilde{y}_\tau\right\}_{\tau=1}^{t}$.

GAUSS resultat for FTSE100 / Equation

Missing cases:	[illegible]	Deletion method:	
Total SS:	[illegible]	Degrees of freedom	
R-squared:	[illegible]	Rbar-squared:	
Residual SS:	[illegible]	Std error of est:	
F(104, 249875):	[illegible]	Probability of F:	

Variable	Estimate	Standard Error	t-value	Prob >|t|	Standardized Estimate	Cor with Dep Var
CONSTAN	-0.01774	0.001111	-15.973021	0		
X001	0.942077	0.002116	445.222785	0		
X002	0.004798	0.002895	1.657381	0.097		
X003	-0.00859	0.002895	-2.967714	0.003		
X004	0.006659	0.002895	2.300231	0.021		
X005	-0.00086	0.002895	-0.298334	0.765		
X006	-0.00273	0.002895	-0.941584	0.346		

Variable	Estimate	Standard Error	t-value	Prob >|t|
CONSTAN	0.052378	0.001869	28.031778	0
X001	0.793858	0.002026	391.807064	0
X002	-0.00116	0.002575	-0.4491	0.653
X003	-0.00063	0.002575	-0.245135	0.806
X004	0.000498	0.002575	0.193579	0.847
X005	0.002442	0.002575	0.948558	0.343
X006	-0.00236	0.002575	-0.91727	0.359

X090	-0.00158	0.002655	-0.596063	0.551
X091	-0.00202	0.002655	-0.761079	0.446
X092	0.001711	0.002655	0.644309	0.519
X093	-0.0016	0.002655	-0.602165	0.547
X094	-4.1E-05	0.002655	-0.015331	0.988
X095	0.003773	0.002655	1.420874	0.155
X096	-0.00026	0.002655	-0.098437	0.922
X097	0.001978	0.002655	0.745089	0.456
X098	-0.0017	0.002655	-0.638479	0.523
X099	-0.00213	0.002655	-0.801291	0.423
X100	-0.00276	0.002655	-1.037646	0.299
X101	0.002768	0.002655	1.042713	0.297
X102	0.000717	0.002655	0.270149	0.787
X103	0.000878	0.002591	0.338832	0.735
X104	-0.0018	0.001718	-1.050138	0.294

X090	0.000977	0.001637	[illegible]	[illegible]
X091	[illegible]	0.001637	-1.076088	0.282
X092	[illegible]	0.001637	-0.32799	0.743
X093	-6.6E-05	0.001637	[illegible]	0.949
X094	[illegible]	0.001637	-0.373671	0.709
X095	[illegible]	0.001637	1.873864	0.061
X096	-0.00045	0.001637	-0.429424	0.668
X097	0.000178	0.001637	0.171674	0.864
X098	-5.5E-05	0.001637	-0.053329	0.958
X099	[illegible]	0.001637	-0.129124	0.897
X100	-0.00003	0.001637	-0.28436	0.776
X101	0.001005	0.001637	0.969881	0.332
X102	-0.00038	0.001036	-0.364084	0.716
X103	0.000277	0.001031	0.219651	0.826
X104	-0.00019	0.000637	-0.296386	0.767

SV models predictive relevance

The Kalman filter:

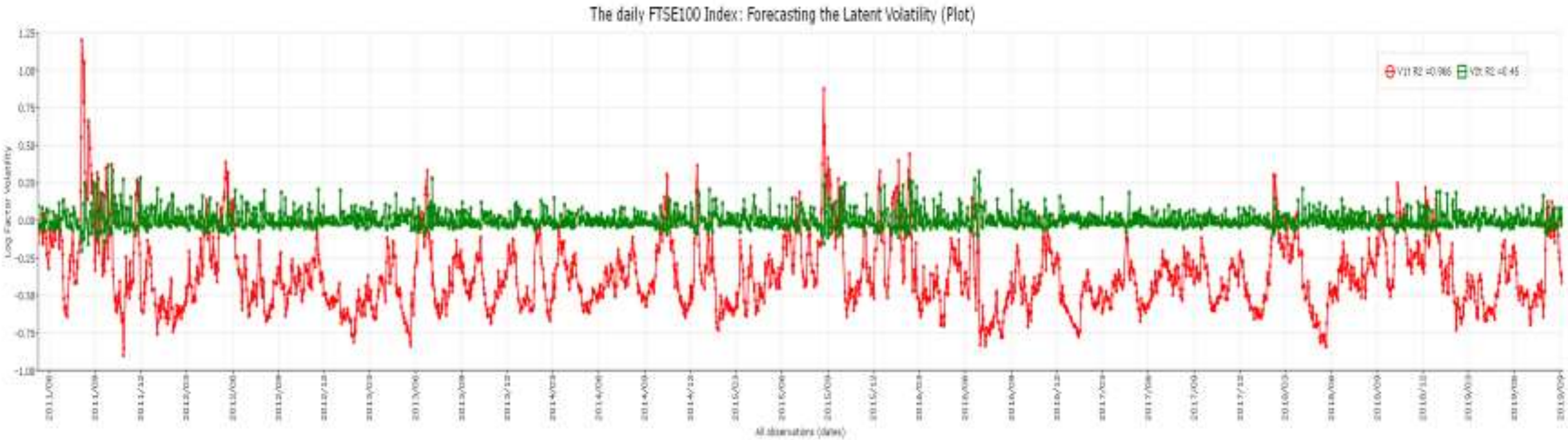
Step 3: Evaluate the SV-model functions on the observed data series $\left\{\tilde{y}_\tau\right\}_{\tau=1}^{t}$, which gives predicted values for $\tilde{V}_{i,t}, i=1,2$ at the actual observed data points.

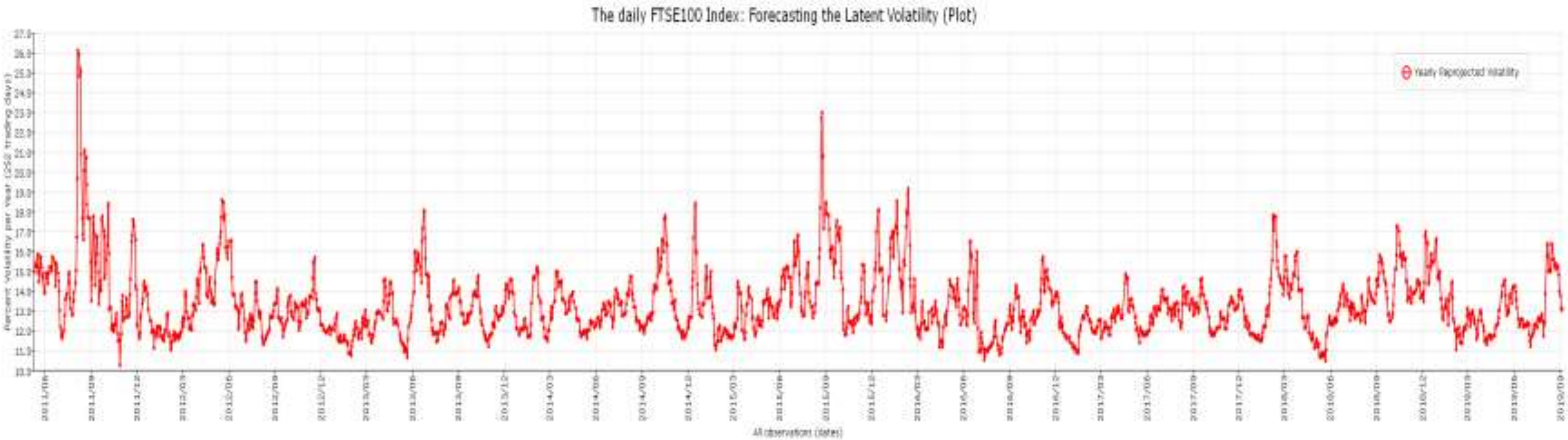
SV models predictive relevance

Date	Prices	y(t)	Var(yt)	Year	V1t R2 =0.966	V2t R2 =0.45	Reprojected Volatility	Reprojected Per Year
20110406	6041.13	0.565563638						
20110407	6007.37	-0.560403176						96.60%
20110408	6055.75	0.802118513						45.00%
…………………… FTSE100 Index								
20190715	7531.72	0.342473148	0.290157919	8.551011382	-5.17E-01	1.58E-02	0.605830426	12.35594056
20190716	7577.2	0.602030289	0.280405993	8.406087687	-5.67E-01	4.12E-02	0.590803047	12.20173626
20190717	7535.46	-0.552385962	0.272070422	8.280202078	-4.99E-01	-3.08E-02	0.588731892	12.18032991
20190718	7493.09	-0.563861614	0.324884261	9.048250313	-4.31E-01	-3.25E-02	0.62893243	12.58931978
20190719	7508.7	0.208108574	0.373307084	9.699143532	-4.51E-01	1.30E-03	0.637517068	12.67494777
20190722	7514.93	0.08293602	0.35147869	9.411303309	-4.60E-01	-1.24E-02	0.623403107	12.53385746
20190723	7556.86	0.556405154	0.332820622	9.158100056	-5.11E-01	3.41E-02	0.620668483	12.5063367
20190724	7501.46	-0.735809173	0.316872424	8.935986288	-4.24E-01	-3.64E-02	0.631053332	12.61052892
20190725	7489.05	-0.165571456	0.413251847	10.20487459	-4.08E-01	-2.38E-02	0.649612096	12.79461794
20190726	7549.06	0.798109849	0.388729805	9.897469928	-4.82E-01	5.96E-02	0.655231552	12.84983856
20190729	7686.61	1.805680187	0.364661437	9.586171403	-6.41E-01	1.62E-01	0.619211	12.49164408
20190730	7646.77	-0.519651762	0.344088725	9.311839703	-5.72E-01	-2.83E-02	0.548544649	11.75726378
20190731	7586.78	-0.787607749	0.37853539	9.766827446	-4.73E-01	-3.70E-02	0.600265713	12.29906337
20190801	7584.87	-0.02517854	0.483605217	11.03940735	-4.69E-01	-1.82E-02	0.614399464	12.44301671
20190802	7407.06	-2.372187412	0.445757277	10.59862415	-1.95E-01	-7.58E-02	0.762904193	13.86549158
20190805	7223.85	-2.504554766	1.624192868	20.23108012	7.77E-02	-9.23E-02	0.985535667	15.7592826
20190806	7171.69	-0.724672062	2.764551045	26.39444759	1.23E-01	-5.72E-02	1.067742613	16.40338802
20190807	7198.7	0.3759123	2.504012424	25.11993493	4.69E-02	4.36E-03	1.052547984	16.2862547
20190808	7285.9	1.204051938	2.172723389	23.39927978	-9.67E-02	8.99E-02	0.993221484	15.82061357
20190809	7253.85	-0.440861114	1.889550307	21.82124372	-7.35E-02	-4.06E-02	0.89216674	14.9941995
20190812	7226.72	-0.374709445	1.684149664	20.60110956	-5.55E-02	-4.02E-02	0.908741858	15.13284336
20190813	7250.9	0.334033114	1.498115068	19.4300025	-1.13E-01	2.30E-03	0.895644209	15.02339312
20190814	7147.88	-1.430978985	1.312921148	18.18945104	2.61E-02	-6.08E-02	0.965862007	15.60119308
20190815	7067.01	-1.137833251	1.591427617	20.02597712	1.23E-01	-6.10E-02	1.064394132	16.377647
20190816	7117.15	0.706988766	1.667229368	20.49736082	2.15E-02	3.54E-02	1.058544643	16.33258247
20190819	7189.65	1.013512744	1.457473525	19.16463744	-1.02E-01	7.06E-02	0.968858005	15.62537095
20190820	7125	-0.903276567	1.278182325	17.94719883	-2.44E-02	-4.92E-02	0.929028003	15.30081883
20190821	7203.97	1.10225368	1.293417497	18.05384195	-1.47E-01	8.22E-02	0.937459596	15.37009493
20190822	7128.18	-1.057632058	1.137953563	16.93411639	-4.38E-02	-5.02E-02	0.910298595	15.14579962
20190823	7094.98	-0.466845065	1.238776	17.66837718	-1.76E-02	-4.57E-02	0.938692245	15.38019655
20190827	7089.58	-0.07613913	1.13369762	16.90241995	-4.22E-02	-3.46E-02	0.926122841	15.27687651
20190828	7114.71	0.353837133	1.001437971	15.88591731	-1.01E-01	1.18E-02	0.914391597	15.17981168
20190829	7184.32	0.973640169	0.888380733	14.96235091	-2.11E-01	7.49E-02	0.872442597	14.82752624
20190830	7207.18	0.317687791	0.791743768	14.12513467	-2.56E-01	6.94E-03	0.779528014	14.01574327
20190902	7281.94	1.031955888	0.70914222	13.36801553	-3.62E-01	7.94E-02	0.753551628	13.78023985
20190903	7268.19	-0.189001804	0.638537604	12.68508873	-3.51E-01	-2.77E-02	0.684647924	13.13511617
20190904	7311.26	0.590833334	0.582994487	12.12083375	-4.14E-01	3.51E-02	0.684954594	13.1380[illegible]761
20190905	7271.17	-0.549841158	0.53071139	11.56456962	-3.55E-01	-3.36E-02	0.677829513	13.06954618
20190906	7282.34	0.153502519	0.54537787	11.72327699	-3.77E-01	-6.82E-03	0.681110687	13.10114091

SV models predictive relevance

FTSE100 Index



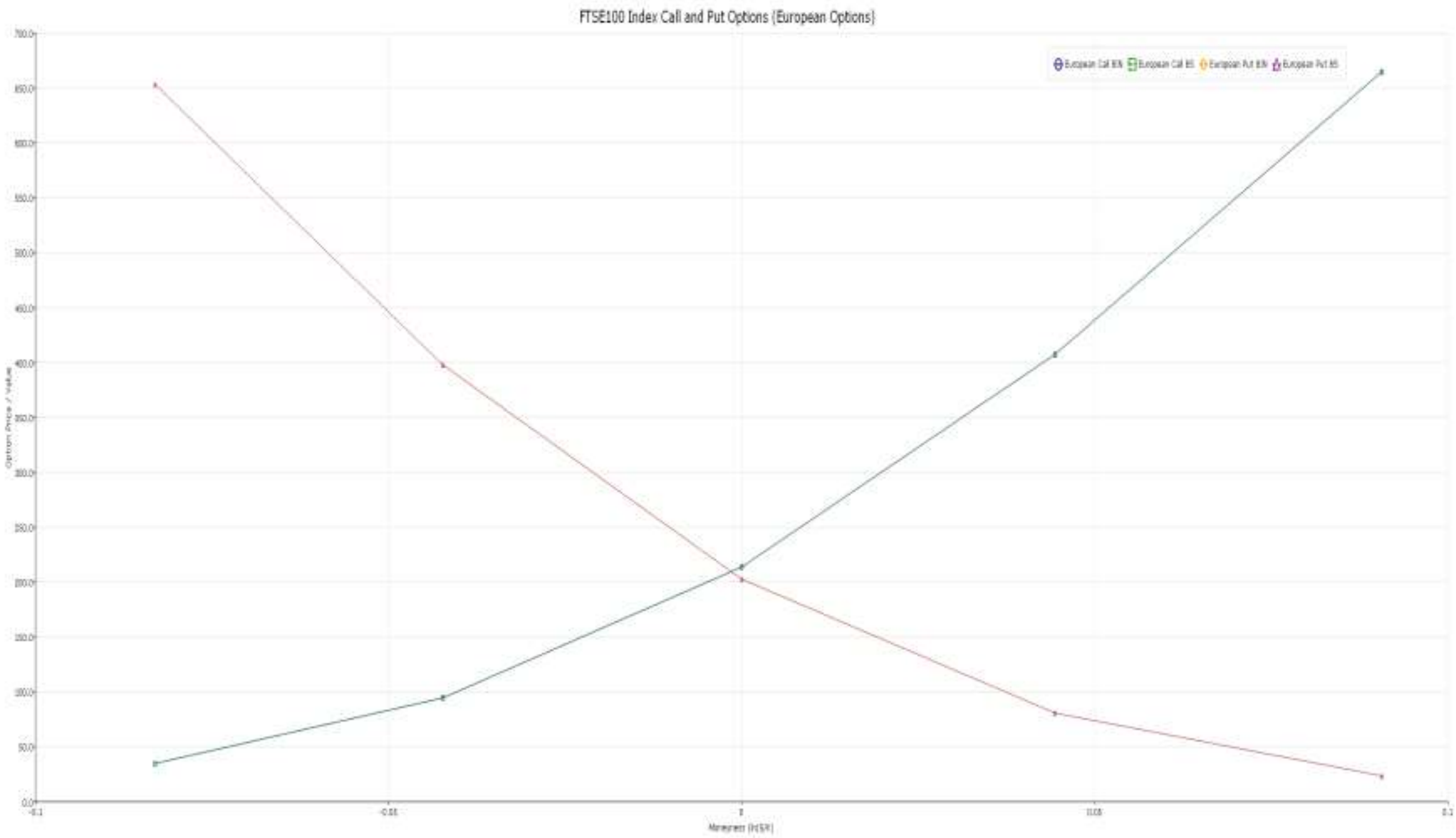


SV models predictive relevance

FTSE100 Index

SV models predictive relevance

Option price calculations:	
K-Calculations	0.13



theta BS
Vega BS
rho BS

SV models predictive relevance

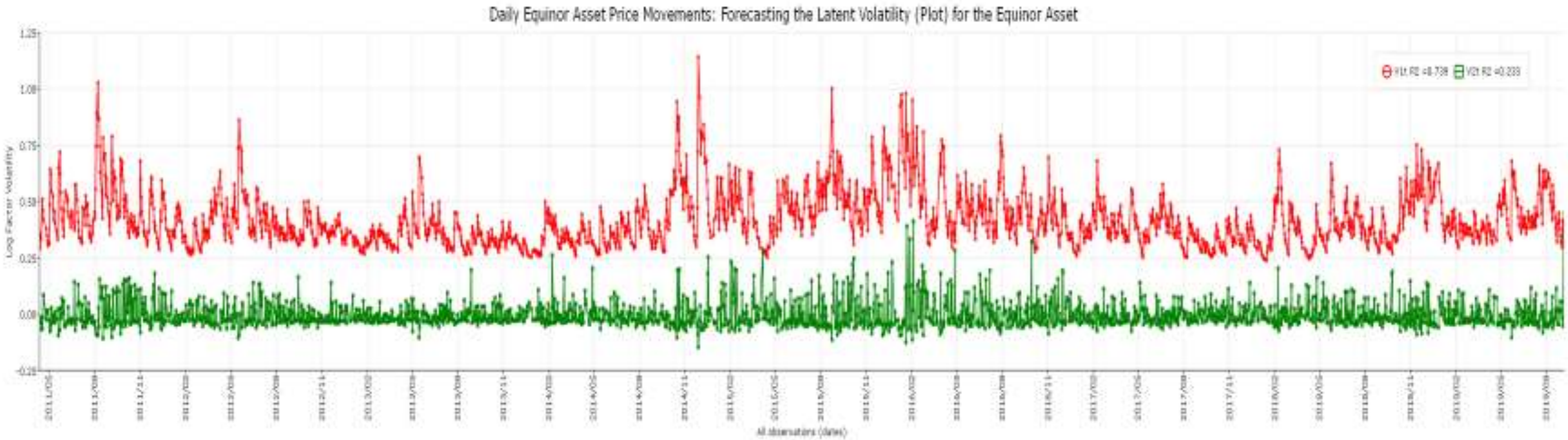
Date	Prices	yt	Var(yt)	Year	V1t R2 =0.739	V2t R2 =0.233	Reprojected Volatility	Reprojected Per Year
20110315	143.9	-2.131397289						
20110316	148.1	2.876910738						73.90%
20110317	152.5	2.927687477						23.30%

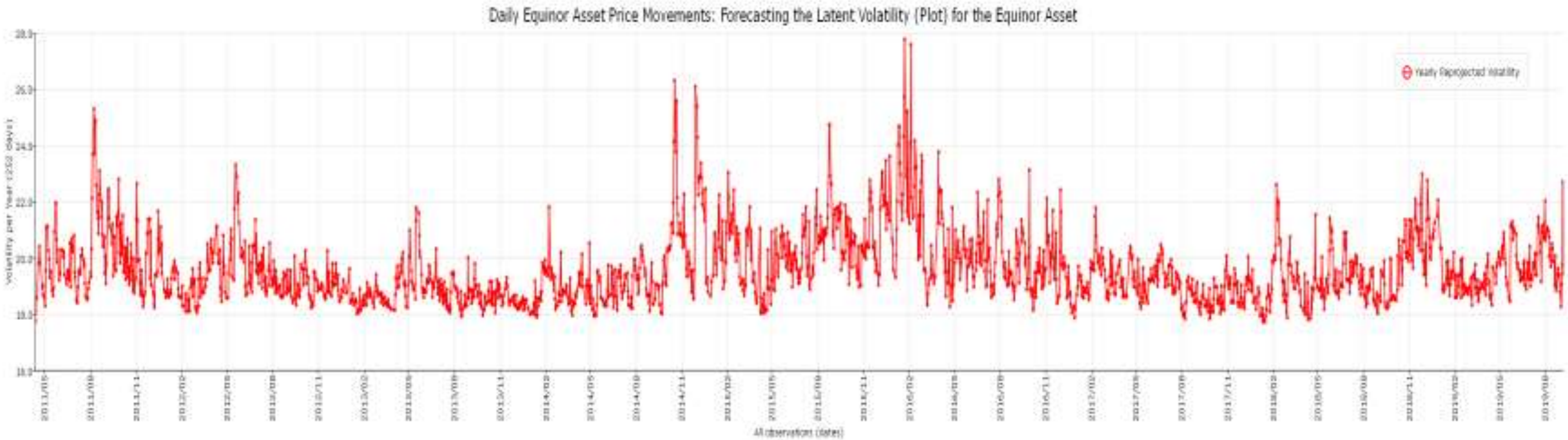
…………………….. Equinor asset

20190715	171.15	-0.495411184	2.15463013	23.30164786	4.36E-01	-3.25E-02	1.497138232	19.42366687
20190716	167.7	-2.036369615	1.929036432	22.04806524	5.32E-01	-5.47E-02	1.612410324	20.15756438
20190717	163.65	-2.444666788	2.529867566	25.24928963	6.41E-01	-6.34E-02	1.780892954	21.18454683
20190718	161.3	-1.446401582	3.048968652	27.7189484	6.59E-01	-5.50E-02	1.828628842	21.46658958
20190719	162.1	0.494744361	2.714152602	26.15275236	5.76E-01	-7.42E-03	1.765983563	21.09568339
20190722	163.15	0.64565943	2.422688759	24.70865369	5.08E-01	7.68E-03	1.674967258	20.5448716
20190723	165.1	1.188132789	2.347680029	24.32314468	4.48E-01	3.63E-02	1.622632713	20.22136108
20190724	165.2	0.060551016	2.375779938	24.46827628	4.00E-01	-2.24E-02	1.459442026	19.1775752
20190725	158.65	-4.045634306	2.143195242	23.23973324	6.35E-01	-8.13E-02	1.740043573	20.94017623
20190726	160.15	0.941035803	4.750224945	34.59850699	5.62E-01	1.15E-02	1.773895864	21.14288906
20190729	158	-1.351584303	3.044071229	27.69667759	5.81E-01	-4.78E-02	1.704890535	20.72757619
20190730	155.15	-1.820264213	3.133322762	28.09977466	6.38E-01	-5.59E-02	1.789244216	21.2341598
20190731	158.9	2.388268265	3.39432381	29.24670238	5.63E-01	9.19E-02	1.925349009	22.02698232
20190801	157.1	-1.139252828	3.583542673	30.05083615	5.67E-01	-4.43E-02	1.68742586	20.62113762
20190802	155	-1.345742834	3.084581234	27.88035995	5.87E-01	-4.81E-02	1.713438996	20.7794761
20190805	152.35	-1.724461149	3.144258532	28.14876818	6.25E-01	-5.58E-02	1.766959686	21.10151276
20190806	151.1	-0.823863615	3.349137284	29.05137855	6.01E-01	-4.17E-02	1.748838874	20.99303209
20190807	149.35	-1.164932462	3.043249198	27.69293769	6.01E-01	-4.80E-02	1.739219368	20.93521628
20190808	150.45	0.733825841	3.086030879	27.88691058	5.31E-01	6.27E-03	1.71075428	20.76319047
20190809	149.75	-0.466356608	2.882478133	26.9515211	5.02E-01	-3.31E-02	1.598536403	20.07065454
20190812	149.4	-0.23399643	2.746724263	26.30920968	4.65E-01	-3.17E-02	1.541811207	19.7113273
20190813	152.45	2.020940074	2.619672625	25.69353034	4.18E-01	7.74E-02	1.64144874	20.33826646
20190814	147.95	-2.996229459	2.979531319	27.40149435	5.73E-01	-6.14E-02	1.667105188	20.49659746
20190815	147.25	-0.474255631	3.916714818	31.41674926	5.38E-01	-4.14E-02	1.642675123	20.34586275
20190816	147.4	0.101815722	2.92380439	27.14403629	4.76E-01	-2.33E-02	1.573384845	19.9121315
20190819	148.7	0.878087371	2.779513527	26.46577807	4.30E-01	1.96E-02	1.567002965	19.8717072
20190820	148.9	0.134408622	2.750790264	26.32867537	3.83E-01	-1.94E-02	1.438983243	19.04268304
20190821	153	2.71629817	2.564073499	25.4194123	3.56E-01	1.14E-01	1.599450557	20.07639261
20190822	154.15	0.748823294	3.317561418	28.91410516	3.22E-01	2.14E-02	1.40953359	18.84681577
20190823	151.7	-1.602126798	2.529705762	25.24848217	4.01E-01	-4.42E-02	1.42945798	18.97955244
20190826	149.9	-1.193648125	2.797868291	26.55301884	4.49E-01	-4.45E-02	1.498697744	19.43378068
20190827	151	0.731143171	2.685333158	26.01353409	4.10E-01	1.28E-02	1.525669598	19.6078744
20190828	152.3	0.857242309	2.493428966	25.06679276	3.75E-01	2.12E-02	1.486515182	19.35463319
20190829	154.65	1.531223923	2.42448751	24.71782459	3.47E-01	5.59E-02	1.496701268	19.4208321
20190830	155.8	0.740863431	2.527992741	25.23993207	3.21E-01	1.70E-02	1.401578513	18.79355702
20190902	155.85	0.032087278	2.261779633	23.87401239	2.98E-01	-1.70E-02	1.324533197	18.2697117
20190903	152.65	-2.074628703	2.117522647	23.10012353	4.18E-01	-5.18E-02	1.442131735	19.06350433
20190904	155.1	1.5922351	2.75332936	26.34082381	3.85E-01	5.57E-02	1.553340392	19.784[illegible]764
20190905	166.75	7.242561461	2.506953857	25.13468464	3.63E-01	3.52E-01	2.044971338	22.70094221
20190906	163.45	-1.998855163	8.134033133	45.27445582	4.49E-01	-4.55E-02	1.497240806	19.42433224

SV models predictive relevance

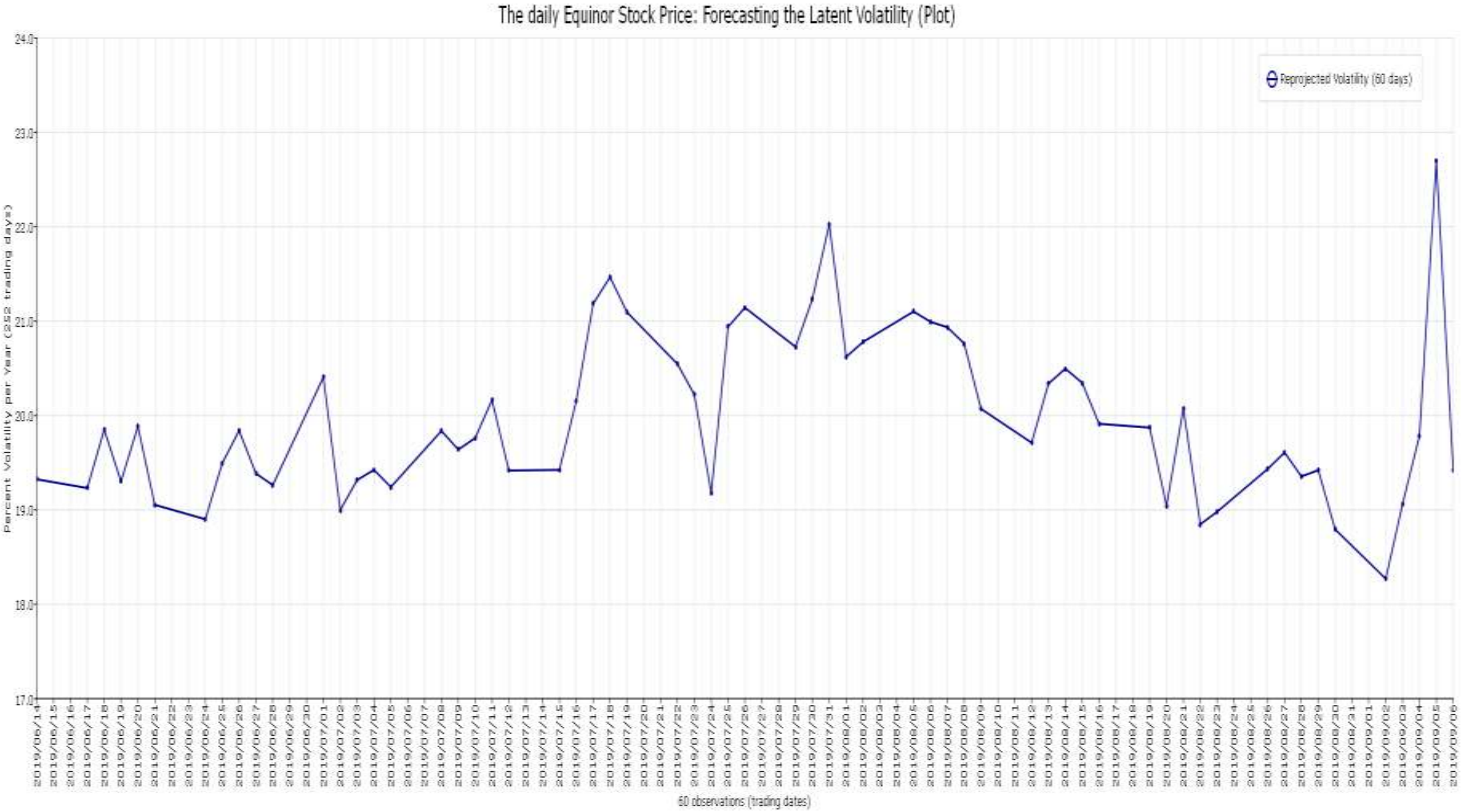
Equinor Asset





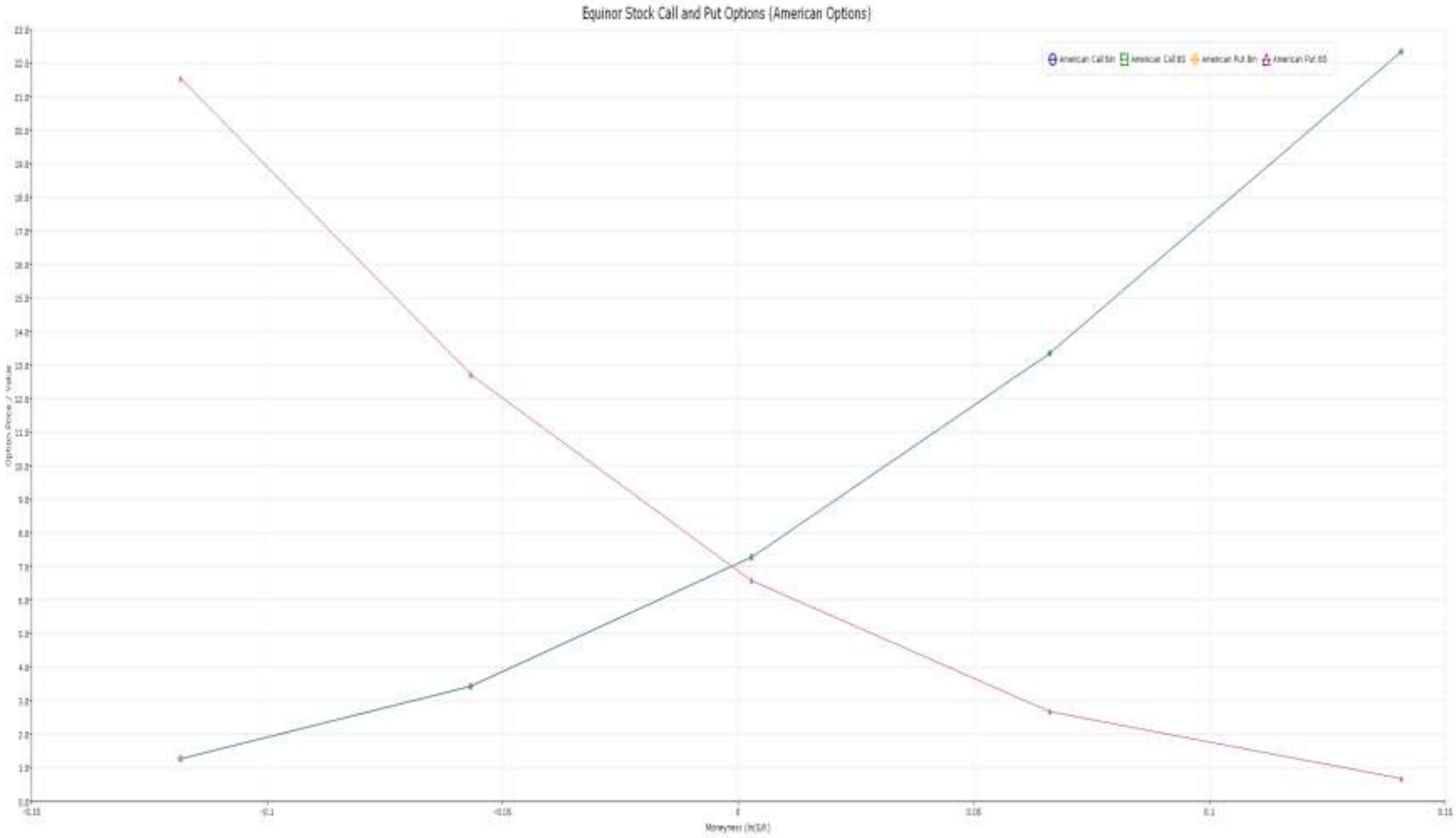
SV models predictive relevance

Equinor Asset



SV models predictive relevance

Option price calculations:
K-Calculations 0.19



Vega BS	13.600589	28.182654	35.560603	32.034086	20.670043
rho BS	-4.2413924	-12.8472	-24.670152	-37.136806	-48.249001

SV models predictive relevance

Just as an exercise: *goodness of fit measures for the estimated latent volatility*

Forecasting volatility using static (one-step ahead) forecasts. Insert actuals for each step ahead.

Estimation from start of sample to 01.01.2019 using 20 lags (days).

Forecast 01.01.2019 to end of sample 06.09.2019. Show actuals and report ±2 * S.E.

SV models predictive relevance

Prediction performance evaluation:

Root Mean Square error: $\sqrt{\sum_{t=T+1}^{T+h} \left(\hat{y}_t - y_t\right)^2 / h}$

Mean Absolute Error: $\sum_{t=T+1}^{T+h} \left(\left|\hat{y}_t - y_t\right|\right) / h$

Mean Absolute Percent Error: $100 \cdot \sum_{t=T+1}^{T+h} \left(\left|\frac{\left|\hat{y}_t - y_t\right|}{y_t}\right|\right) / h$

Symmetric MAPE: 200 multiplied by the mean of the absolute difference between $\hat{y}$ and *y* divided by the sum of the absolute of $\hat{y}$ and *y*.

SV models predictive relevance

Performance evaluation methods(1):

Theil Inequality Coef. (U1)

(RMSE normalized by the dispersion of actual and forecasted series)

$$\frac{\sqrt{\sum_{t=T+1}^{T+h}\left(\hat{y}_t - y_t\right)^2 / h}}{\sqrt{\sum_{t=T+1}^{T+h}\hat{y}_t^{\,2} / h} + \sqrt{\sum_{t=T+1}^{T+h}\hat{y}_t^{\,2} / h}}$$

Bias Proportion:

$$\frac{\left(\left(\sum \hat{y}_t / h\right) - \bar{y}\right)^2}{\sum\left(\hat{y}_t - y_t\right)^2 / h}$$

Variance Proposition:

$$\frac{\left(s_{\bar{y}} - s_y\right)^2}{\sum\left(\hat{y}_t - y_t\right)^2 / h}$$

Covariance Proportion:

$$\frac{2\left(1 - r\right)s_{\hat{y}}s_y}{\sum\left(\hat{y}_t - y_t\right)^2 / h}$$

SV models predictive relevance

Performance evaluation method (2):

Theil Inequality Coef. (U2)

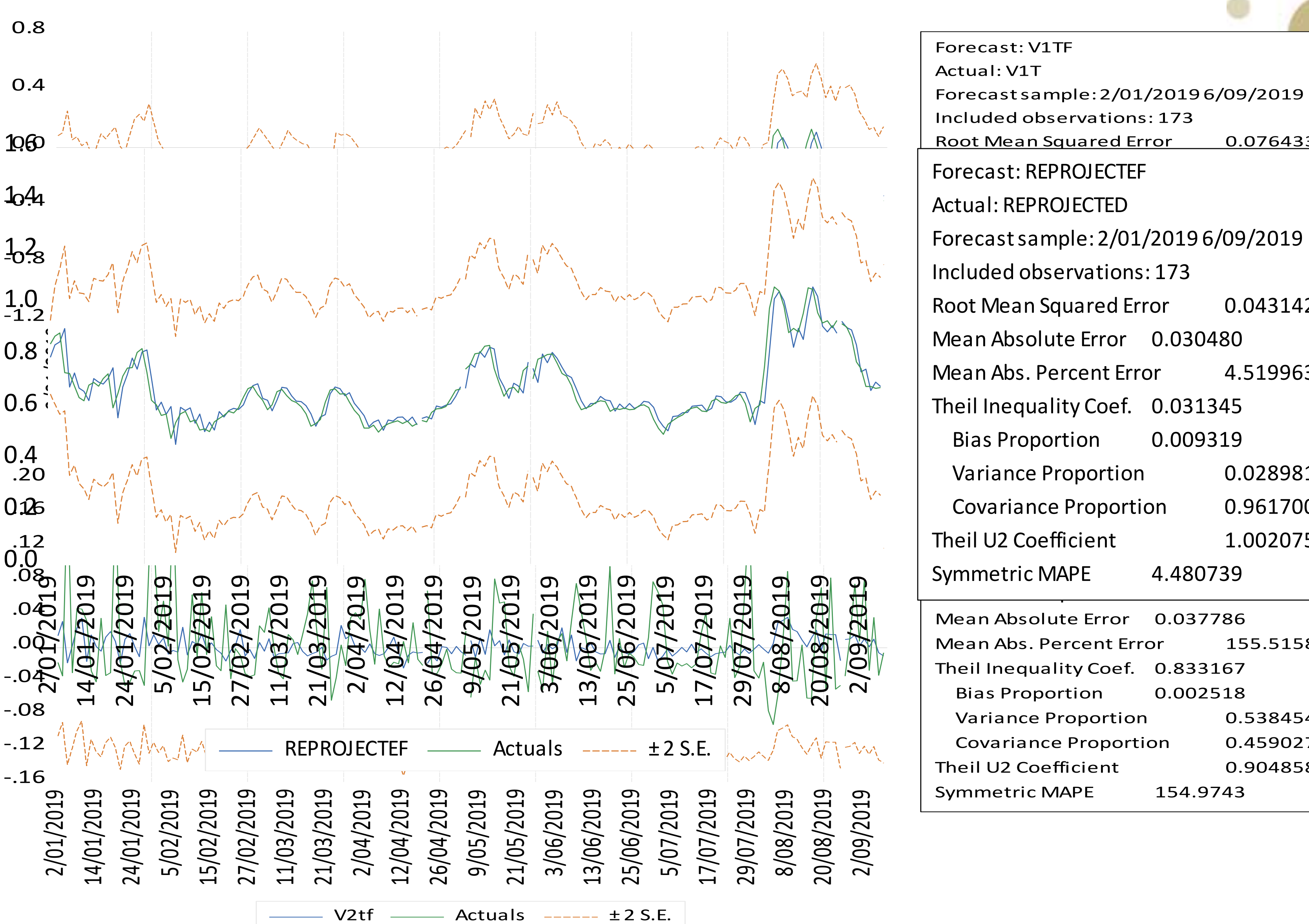
(RMSPE relative to naive forecast)

$$\sqrt{\frac{\sum_{t=T+1}^{T+h}\left(\frac{\hat{y}_t - y_t}{y_{t-1}}\right)^2 / h}{\sum_{t=T+1}^{T+h}\left(\frac{y_{t-1} - y_t}{y_{t-1}}\right)^2 / h}}$$

If your forecast is “good”, the bias and variance proportions should be small so that most of the bias should be concentrated on the covariance proportions (U1).

For additional discussion of forecast evaluation, see Pindyck and Rubinfeld (1998, p. 210-214).

SV models predictive relevance



SV models predictive relevance

Forecast: V1TF
Actual: V1T
Forecast sample: 2/01/2019 6/09/2019
Included observations: 171

Mean Absolute Error	0.038762
Mean Abs. Percent Error	112.4466
Theil Inequality Coef.	0.867378
Bias Proportion	0.003537
Variance Proportion	0.734898
Covariance Proportion	0.261565
Theil U2 Coefficient	0.922842
Symmetric MAPE	162.2093

Forecast: REPROJECTEF
Actual: REPROJECTED
Forecast sample: 2/01/2019 6/09/2019
Included observations: 171

Root Mean Squared Error	0.099294
Mean Absolute Error	0.071158
Mean Abs. Percent Error	4.602450
Theil Inequality Coef.	0.032387
Bias Proportion	0.002214
Variance Proportion	0.063128
Covariance Proportion	0.934658
Theil U2 Coefficient	0.925288
Symmetric MAPE	4.587966

Thank you!

Professor
Dr.oecon Per B Solibakke

per.b.solibakke@ntnu.no

See also: http://folk.ntnu.no/perbso/ (volatility indices)

NTNU – Trondheim
Norwegian University of
Science and Technology