Chapter VI

Testing the Bivariate Conditional CAPM in Thinly Traded Markets: Controlling for Non-synchronous Trading and Volatility Clustering

Abstract

This paper investigates and tests the conditional Capital Asset Pricing Model (CAPM) in the Norwegian thinly traded market. By applying a bivariate ARMA-GARCH-in-Mean lag specification for the conditional mean and volatility the full covariance matrix is estimated. The full covariance matrix makes it possible to develop alternative mean specifications and consequently to test the conditional CAPM versus among others the residual risk and the one dynamic factor models. Importantly, as the bivariate ARMA-GARCH-in-Mean lag specification accounts for non-synchronous trading and volatility clustering, which may induce serious biases in the moments and the co-moments of asset returns we apply elaborate specification test statistics to see how well the ARMA-GARCH model control for non-synchronous trading and volatility clustering. As non-synchronous trading and non-trading effects increases as trading frequency decreases, we apply trading volume in NOK as a measure of trading frequency for our series. Our results find that the in-Mean specifications are redundant, which imply that we are unable to find any preference among conditional asset pricing models. Non-synchronous trading and non-trading effects are found in all bivariate estimations. Index series show strong positive serial correlation while return series show positive (negative) serial correlation for frequently (thinly) traded series. Positive cross-autocorrelation from index to return series is strongly significant and seems to increase as thin trading increases. Volatility clustering is strong in all bivariate estimation and seems to induce a rejection of the independence hypothesis. We also find that a major force driving the conditional variances of Norwegian return series, is the history contained in the conditional market variance. For especially frequently traded assets the conditional variance is heavily influenced by the past squared market shocks. However, when thin trading becomes severe the specification tests report model misspecification. Finally, due to low market correlation, thinly traded series show both high relative conditional variance and low to negative time varying betas. Our results suggest that the conditional betas cumulative distribution functions classify unambiguously the betas in ascending order of trading volume. However, as thinly traded assets report misspecification, thin trading may induce serious biases to the co-moments.

Classification:

Keywords: ARMA-GARCH-in-Mean, Risk Models, Trading frequency, Non-synchronous trading, volatility clustering

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1 Introduction

This paper makes use of a bivariate ARMA-GARCH-in-Mean specification for the purpose of studying thinly traded market characteristics on empirical applications of the Capital Asset Pricing Model (CAPM). The ARMA-GARCH-in-Mean specification estimates the full covariance matrix and makes it possible to carry through tests of among others the conditional CAPM, the residual risk model and the one dynamic factor model. Earlier studies of market volatility have shown that volatility moves together over time and across assets and markets. Recognising this commonality through a multivariate framework leads to obvious gains of efficiency. In this investigation we focus on trading frequency and therefore how asset mean and volatility moves together across thinly and frequently traded assets and the market. Trading volume in NOK is employed as a proxy for trading frequency. In fact, thin trading characteristics (including non-trading[1]) may induce non-synchronous trading and non-trading effects as well as conditional heteroscedasticity. Therefore, our model specifications employ a bivariate ARMA-GARCH lag specification to account optimally of non-synchronous trading effects in the conditional mean and conditional heteroscedasticity in the conditional volatility, by employing the efficient lag specification from the Bayes Information Criterion (BIC) (Schwarz, 1978) in both the conditional mean and volatility. We therefore do not pursue a simultaneous return and trading volume specification as pursued in Clark (1973), Tauchen and Pitts (1983), Gallant et al. (1992) and Andersen, (1994), but rather employ individual return series to study characteristics over a wide variety of trading frequencies optimal ARMA-GARCH-in-Mean specifications. The advantage of such modelling is an explicit availability of the conditional mean and volatility series for individual series and the market index. Hence, the bivariate specification may give new insight to return and volatility characteristics of thinly traded markets. As the Norwegian market is a professional dealer market, it is ideal for this kind of market study as the market is a relatively thinly traded market and contains assets that exhibit relative thin trading frequency.

We design a BIC (Schwarz, 1978) preferred bivariate ARMA-GARCH-in-Mean specification where we pair each individual return series with the index return series. The specification is a bivariate ARMA-GARCH approach (MGARCH) and will be able to capture temporal dependencies in the conditional mean, variances and covariance. The estimation is a one-stage procedure in which betas and risk premium are estimated simultaneously[2]. The bivariate ARMA-GARCH-in-Mean lag specification's error process will assume that the residuals of the regressions should be serially uncorrelated, conditional homoscedastic and normally

[1] Solibakke (2000) show employing autocorrelation characteristics that the Norwegian market is a relatively thinly traded market and contains relatively thinly traded assets.

[2] Applying OLS estimation of CAPM in a time series context, the underlying theory requires a number of assumptions to hold. Specifically, we assume that the risk premiums are stationary, normally distributed and serially uncorrelated, which imply that the error process is i.i.d.

distributed. Hence, we employ residual specification tests for the bivariate ARMA-GARCH-in-Mean model, to report any model misspecifications.

Our results show some interesting features. Firstly, we find a significant positive "zero-beta" coefficient for only the frequently traded series. This result suggests that it is the frequently traded assets that show a significant positive drift. Secondly, all series report significant autocorrelation and cross-autocorrelation. Thinly traded series report negative autocorrelation and frequently traded series report positive autocorrelation. The negative serial correlation in thinly traded series may be induced by non-trading and mean reversion. All asset series report positive cross-autocorrelation from the market index. Hence, frequently traded assets report slow adjustments at time *t* from its own and market past return (*t-1)*. In contrast, thinly traded assets report mean reversion from its own one period lagged returns and slow adjustment to market lagged returns. The mean reversion seem to increase as thin trading increases. These results induce return predictability for both frequently and thinly traded assets. However, thinly traded series may show spurious autocorrelation due to periods of zero returns. Thirdly, all alternative in-Mean specifications are rejected, which imply rejection of the residual risk hypothesis, the one dynamic factor model and the conditional CAPM as well as no preference among alternative risk measures. This result suggests that the in-Mean model is not a very well specified risk and the volatility feedback methodology. Fourthly, conditional heteroscedasticity is present in all asset and market index series and the univariate modelling approach of the conditional variances seem to be rejected due to significant bivariate GARCH coefficients. The market dynamics may therefore not be adequately comtrolled for in an univariate modelling approach. Fifthly, asymmetric volatility seems to be present in almost all series. Sixthly, specification test statistics report data-dependence for thinly traded assets and suggest biases in the moments and co-moments of the asset series. The autocorrelation and cross-autocorrelation result for thinly traded assets may therefore be spurious and the predictability may depend on non-trading effects rather than return predictability. Seventhly, the data dependence result for the thinly traded assets suggests a rejection of the ARMA-GARCH specification. Severe non-synchronous trading and non-trading may therefore suggest ARMA-GARCH misspecification.

Finally, as we estimate the full co-variance matrix the conditional beta (β) measure is readily available. Our results show that the conditional CAPM's β series cumulative frequency distribution, classify all series in nicely ascending order of trading frequency. Hence, trading frequency seems to classify the relevant risk measure[3]. However, as co-moments are biased for thinly traded series the result must be interpreted by caution.

[3] Due to high trading frequency correlation with market value, the relevant risk measure may also be classified also in accordance with size.

The rest of the article is organised as follows. Section 2 defines the methodology. Section 3 presents the data and adjustment procedures for stationery data series. Section 4 reports the results/findings of the analysis. Section 5 reports the conditional (co-) variance and beta characteristics and Section 6 summarises and concludes.

2 Methodology

2.1 The Static CAPM (the Sharpe-Lintner-Black CAPM)

Let R_i denote the return on any asset *i* and R_M be the return on the market index (*M*) return of a value weighted asset index in the economy[4]. The Black (1972) version of the CAPM is $E[R_i] = \mu_0 + \delta_1 \cdot \beta_i$, where β_i is defined as $\beta_i = Cov(R_i, R_M) / Var[R_M]$, and $E[\cdot]$ denotes the expectation, $Cov(\cdot)$ denotes the covariance and $Var[\cdot]$ denotes the variance. Fama and French (1992) finds that the estimated value for δ_1 is close to zero and concludes therefore that the results suggest strong evidence against the CAPM. The static CAPM is also tested in the Norwegian market. However, Carlsen og Ruth (1991) fail to the reject the null of μ_0 significant different from zero for both univariate and multivariate tests[5]. The empirical results seem therefore to show no clear evidence for or against the static CAPM. However, this result does not necessarily imply evidence for or against the conditional CAPM. The CAPM was developed within the framework of a hypothetical single-period model economy. The real world, however, is dynamic and hence, expected returns and betas are likely to vary over time. Even when expected returns are linear in betas for every time period based on the information available at that time, the relation between the unconditional expected returns and the unconditional beta could be close to zero[6]. In the next section we assume that CAPM holds in a conditional sense, i.e., it holds at every point in time, based on whatever information is available at that instant.

2.2 The Conditional CAPM

If we assume that expectations in CAPM at time *t* are conditioned on the information set available to agents at time *t-1*, Ω_{t-1}, then the conditional CAPM[7] can be written as $E_t(R_{i,t} \mid \Omega_{t-1}) = \mu_{0,t-1} + \delta_{1,t-1} \cdot \beta_{i,t-1}$, where $\beta_{i,t-1}$ is the conditional beta of asset *i* defined as

$$\beta_{i,t-1} = \frac{Cov(R_{i,t}, R_{M,t} | \Omega_{t-1})}{Var(R_{M,t} | \Omega_{t-1})}.$$

$\mu_{0,t-1}$ is the conditional expected return on a "zero-beta" portfolio, and $\delta_{1,t-1}$ is the conditional market risk premium. Both the expected returns and the

[4] We assume here that the market index is a good approximation for the market portfolio.
[5] See also Carlsen and Ruth, 1990, Stange , 1989, Semmen, 1989 and Hatlen et al. 1988.
[6] Because an asset that is on the conditional mean-variance frontier need not be on the unconditional frontier (Dybvig and Ross, 1985 and Hansen and Richard, 1987)

betas, will in general, be time varying in the conditional CAPM framework. The model is stated in terms of conditional moments and assumes that investors use information at time *t-1* rationally and maximise their utility period by period.

As the model is now stated it is not operational because of the lack of an observed series for the expected market excess return. However the conditional CAPM model assumes neither the beta nor the risk premium is to be constant over time. Hence, if we now reformulate the conditional CAPM and write

$E_t(R_{i,t} \mid \Omega_{t-1}) = \mu_{0,t-1} + Cov(R_{i,t}, R_{M,t} \mid \Omega_{t-1}) \cdot \frac{\delta_{1,t-1}}{Var(R_{M,t} \mid \Omega_{t-1})}$, then we have defined the ratio between the conditional risk premium and the conditional variance of the market portfolio. This ratio, defined as the aggregate risk aversion coefficient λ, can be assumed constant over the sample time periods. Therefore, a testable version of the conditional CAPM is given by the specification

$$E_t(R_{i,t} \mid \Omega_{t-1}) = \mu^i_{0,t-1} + \lambda_{i,t-1} \cdot Cov(R_{i,t}, R_{M,t} \mid \Omega_{t-1}) \qquad (1)$$

where $\mu^i_{0,t-1}$ is the conditional expected return for asset *i*. The model (1) requires the specification of the dynamics of $Cov(R_{i,t},R_{M,t} \mid \Omega_{t-1})$.

2.3 The ARMA-GARCH-in-Mean specification of the Conditional CAPM

Model (1) is possible to write as $R_{i,t} = \mu_{i,0} + \lambda_i \cdot Cov(R_{i,t}, R_{M,t} \mid \Omega_{t-1}) + u_{i,t}$ and

$R_{M,t} = \mu_{M,0} + \lambda_M \cdot Var(R_{M,t} \mid \Omega_{t-1}) + u_{M,t}$, where $\mu_{i,0}$ and $\mu_{M,0}$ is the drift in asset *i* and the market index (*M*), respectively, the $u_{i,t} = R_{i,t} - E_t(R_{i,t} | \Omega_{t-1})$ and $u_{M,t} = R_{M,t} - E_t(R_{M,t} | \Omega_{t-1})$ are the residual terms for asset *i* and the market index (*M*), respectively. We thus see that $Var_t(R_{i,t} | \Omega_{t-1}) = E_t(u^2_{i,t} | \Omega_{t-1}) = h_{i,t}$, and $Var_t(R_{M,t} | \Omega_{t-1}) = E_t(u^2_{M,t} | \Omega_{t-1}) = h_{M,t}$, and $Cov_t(R_{i,t},R_{M,t} | \Omega_{t-1}) = E_t(u_{i,t} \cdot u_{Mt} | \Omega_{t-1}) = h_{i,M,t}$. Moreover, the Norwegian market is a thinly traded market showing non-synchronous trading and non-trading effects. Campbell (1997) show that non-trading potentially induces serious biases in the moments and co-moments of return series such as their mean, variances, covariance, betas and autocorrelation coefficients. Non-synchronous trading and non-trading may be modelled in the ARMA specifications where the lagged length is determined applying the BIC criterion (Schwarz, 1978). Another important feature we find in financial markets are volatility clustering or changing volatility. The observed conditional hetroscedasticity potentially influenced by non-synchronous trading in the conditional mean may be modelled by a GARCH conditional volatility specification. Hence, this time varying conditional CAPM can be put into

[7] See Jagannathan and Wang, 1996.

bivariate ARMA-GARCH-in-Mean form[8] [9]. We apply the following specifications for our CAPM tests and to control for non-synchronous trading and volatility clustering.

$$R_{i,t} = \mu_{i,0} + \sum_{j=1}^{p_i} \phi_{i,j} \cdot R_{i,t-j} + \lambda_i \cdot Cov(R_{i,t}, R_{M,t} \mid \Omega_{t-1}) + \varepsilon_{i,t} - \sum_{j=1}^{q_i} \theta_{i,j} \cdot \varepsilon_{i,t-j} \qquad (2)$$

$$R_{M,t} = \mu_{M,0} + \sum_{j=1}^{p_M} \phi_{M,j} \cdot R_{M,t-j} + \lambda_M \cdot Var(R_{M,t} \mid \Omega_{t-1}) + \varepsilon_{M,t} - \sum_{j=1}^{q_M} \theta_{M,j} \cdot \varepsilon_{M,t-j} \qquad (3)$$

$$\gamma_{k,j,t} = \delta_{k,j} \quad \text{if and only if} \quad \varepsilon_{k,t-j} < 0 \qquad k = i, M \qquad (4)$$

$$h_{k,t} = m_{k,0} + \sum_{j=1}^{r_k} (a_{k,j} + \gamma_{k,j,t}) \cdot u_{k,t-j}^2 + \sum_{j=1}^{s_k} b_{k,i} \cdot h_{k,t-j} \qquad k = i, M \qquad (5)$$

where $\lambda_{i,t-1} = \dfrac{E_t(r_M|\Omega_{t-1})}{Var_t(r_M|\Omega_{t-1})}$. The bivariate system of random vectors $R_t' = (R_{i,t}, R_{M,t})$ followed by the conditional variance-covariance matrix $h_{k,t}$, allows for a rich structure permitting interaction effects between the market index and the individual assets. The a_k are the vectors of the weights for the lagged ε^2 terms; this is the ARCH process. The b_k are the weights for the lagged h_k terms; this is the GARCH process. The m_k is a constant term for unexplained conditional variance. To determine the lag lengths in the conditional variance equation *rk* and *sk*, we apply the BIC criterion (Schwarz, 1978) on the squared residuals from the conditional mean ARMA specification. Solibakke (2000) show the importance of one more important feature of the Norwegian thinly traded market that needs to be incorporated into the bivariate ARMA-GARCH-in-Mean specification. Asymmetric volatility or the “leverage” effect (Nelson, 1991) is specified in the volatility equation (4) as suggested by the GJR model (Glosten et al., 1993). The model therefore apply $\gamma_{k,i,t} = \varepsilon_{k,t-i}$ if and only if $\varepsilon_{k,t-i} < 0$ in the conditional volatility equations. We allow the γ parameter to be less than zero. This theoretical specification of the conditional CAPM provides the central focus of the tests conducted in this paper.

3 Empirical data and adjustment procedures

The study applies daily returns of individual Norwegian stocks spanning the period from October 1983 to February 1994. As some of these assets exhibit thin trading characteristics, the assets are sorted from frequently traded assets (no. 1) to thinly traded assets (no. 7), where trading volume is employed as a proxy for trading frequency. Trading volume is the amount traded in the asset in NOK; that is, the number of stocks traded multiplied by settlement prices at the time of trading. Moreover, individual assets are grouped into portfolios at period *t* based on trading volume at *t-1*. Portfolio FT consists of the most frequently traded assets and Portfolio TT consists of the most thinly traded assets. The portfolio series are

[8] See Hall et al., 1989, Bollerslev et al., 1988, Chan et al., 1992, Gonzales-Rivera, 1996.
[9] For applications see Bollerslev et al., 1992.

rebalanced each month using information at *t-1*. Moreover, assets traded throughout a month, is assigned to one of the two portfolios on basis of their average daily trading volumes in NOK for the last 2 years in the market. The two-year average avoids a too frequent shift of portfolio-assets. To proxy for the market portfolio we employ the value weighted market index[10] consisting of all stocks in the Norwegian market.

The crash in October 1987 is not excluded from the sample series. We therefore assume that a crash is normal in equity markets. Finally, we adjust for systematic location and scale effects (Gallant, Rossi and Tauchen, 1992) in all time series. The log first difference of the price index is adjusted. Let ϖ denote the variable to be adjusted. Initially, the regression to the mean equation $\varpi = x \cdot \beta + u$ is fitted, where *x* consists of calendar variables as are most convenient for the time series and contains parameters for trends, week dummies, calendar day separation variable, month and sub-periods. To the residuals, $\hat{u}$, the variance equation model $\hat{u}^2 = x \cdot \gamma + \varepsilon$ is estimated. Next $\frac{\hat{u}^2}{\sqrt{e^{x \cdot \hat{\gamma}}}}$ is formed, leaving a series with mean zero and (approximately) unit variance given *x*. Lastly, the series $\hat{\varpi} = a + b \cdot (\frac{\hat{u}}{\sqrt{e^{x\hat{\gamma}}}})$ is taken as the adjusted series, where *a* and *b* are chosen so that $\frac{1}{T} \cdot \sum_{i=1}^{T} \hat{\varpi}_i = \frac{1}{T} \cdot \sum_{i=1}^{T} \varpi_i$ and $\frac{1}{T-1} \cdot \sum_{i=1}^{T} (\hat{\varpi}_i - \overline{\varpi})^2 = \frac{1}{T-1} \cdot \sum_{i=1}^{T} (\hat{u}_i - \overline{u})^2$. The purpose of the final location and scale transformation is to aid interpretation. In particular, the unit of measurement of the adjusted series is the same as that of the original series. We do not report the result of these raw data series adjustments[11].

{Insert Table 1 about here}

The characteristics of the assets, the equal weighted trading frequency portfolios and the value weighted market index are reported in Table 1. The following immediate observations can be extracted. The standard deviation of returns seems to increase proportionally with the level of thin trading. The daily maximum and minimum return series seem to suggest that highest absolute numbers are found for the thinly traded series. This variation in mean return among the thinly traded assets produces consequently the highest standard deviation. For the portfolio series the highest absolute minimum is found for the frequently traded series. The portfolio results suggest that thinly traded series containing zero asset returns outweighs high

[10] Note that about 20% of the assets in the Norwegian market count for 60% of the market value of the Oslo Stock Exchange.
[11] The results are readily available from the author upon request.

individual absolute returns. Finally, as expected from the portfolio theory, the market index produces the lowest standard deviation.

The calculated numbers for kurtosis and skew from stock returns, suggest a substantial deviation from the normal distribution. The kurtosis and skew indication of non-normality is strongly supported by the Kolmogorov-Smirnov Z-test statistic[12] (K-S Z-test) for normality for all series. The kurtosis and skew and the K-S Z-test suggest too much probability mass around the mean, too little around 1-2 standard deviation from the mean and some extreme values on especially the negative side of the mean. The results induce that it is the thinly traded series that show the highest deviation from the normal distribution. However, the value-weighted market index reports high kurtosis and a high negative skew. From Table 1 it also seems as especially the kurtosis increases as the number of combined assets increases[13].

The ARCH (Engle, 1982), the RESET (Ramsey, 1969) and the BDS (Brock and Deckert, 1988 and Scheinkman, 1990) test statistics, suggest data dependence in all adjusted return series. The ARCH test suggests changing conditional volatility, which induce conditional heteroscedasticity. The RESET test suggests non-linear dependence in the mean and the BDS test statistic suggests strong general non-linear dependence. Especially where we find long non-trading periods (thin trading), the BDS statistic reports highly significant values. In contrast, the portfolio series report increased non-linear dependence when trading frequency increases, which may stem from a more erratic conditional volatility. Overall the ARCH, RESET and BDS test statistics report surprisingly stable and strongly significant data dependence for all series. Note that a non-linear conditional volatility imply a rejection of the independence hypothesis while a non-linear mean imply a rejection of the Martingale hypothesis.

4 Empirical results

Maximum likelihood estimates[14] of the parameters for the bivariate ARMA-GARCH-in-Mean specification[15] are given in Table 2 for all bivariate asset and market index daily return series. The two intercepts ($\mu_{i,0}$, $\mu_{M,0}$) in the mean equations of the bivariate system of equations are positive for all series. The market index reports significant positive mean drift for all

[12] The K-S Z test statistic is a procedure to test the null that a sample comes from a population in which the variable is distributed according to a normal distribution.
[13] Often named the mixture of distributions hypothesis, which maintains that asset returns exhibit leptokurtosis because they are really a combination of returns distributions.
[14] We assume conditional bi-normality of the residuals. We also employ the BHHH (Berndt et al., 1974) algorithm for maximum likelihood estimation of parameters.
[15] The univariate ARMA lags, determined by the BIC criterion (Schwarz, 1978) are ARMA (0,2) for the thinly traded portfolio and assets no. 4 to 7; ARMA (0,1) for the most frequently traded portfolios and assets no. 1 to 3. All assets and portfolios employ a GARCH (1,1) lag specification applying the BIC criterion on the squared residuals from the ARMA lag

estimations. It also seems that the positive drift is more significant for frequently traded assets. The GARCH-in-Mean parameters (λ) can as outlined above, be specified for several alternative outlines of the conditional means. As we estimate the full variance-covariance matrix in the conditional variance process, the conditional standard deviation, the conditional covariance (with the market portfolio) and the conditional market standard deviation, we can specify several alternative outlines of the conditional means. Firstly, we introduce the conditional variance series (h_i and h_M) in the conditional means for asset and market series, respectively. The conditional variance (h_i) can then be interpreted as residual risk and the accompanying coefficient (λ_i) measures residual risk sensitivity, which is the sensitivity to total risks. The specification may be considered as a proxy for omitted risk factors (Lehmann, 1990). Secondly, we introduce the market variance (h_M) in both conditional means. The introduction of the market variance in the asset mean may be interpreted as a one dynamic-factor model, which implies that the dynamics and variation in the overall market index, guide all the return series. The $\lambda_{i,M}$ coefficient measure the sensitivity to total market dynamics. Thirdly and finally, we run the bivariate estimations with the specification in (2) to (4) in Section 2, which is the conditional CAPM specification.

The results in Table 2 suggest that none of the series report significant in-Mean coefficients (λ). The residual risks specification (λ_i) is rejected. The one dynamic factor hypothesis ($\lambda_{i,M}$) is rejected. Finally, the conditional CAPM specification ($\lambda_{i,i,M}$) is rejected. Moreover, the market index produces insignificant mean coefficient (λ_M) from its own conditional variance process.

Our results report a consistent positive coefficient (θ_i) for thinly traded series and a consistent negative coefficient (θ_i) for frequently traded series. We find significant positive cross-autocorrelation from market index to asset returns ($\phi_{M,i}$). The market index, which is employed as a proxy for the market portfolio, reports strongly significant autocorrelation (θ_M) for all bivariate estimations.

{Insert Table 2 about here}

Panel B of the Tables 2 report the conditional variance equations from the bivariate estimations. The t-statistics indicate that the parameters $m_{i,i}$, $m_{i,M}$, $m_{M,M}$, $a_{i,i}$, $a_{i,M}$, $a_{M,i}$ $a_{M,M,}$ $b_{i,i}$, $b_{i,M}$, $b_{M,i}$, $b_{M,M}$ are almost all statistical significant at conventional levels. Interestingly, the cross-series GARCH parameters show strongly significant values. Asymmetric volatility or the "leverage effect" (γ_i and γ_M) seems to be present in almost all series.

As an overall specification test of the bivariate model, we calculate several elaborate test statistics in Table 3. Firstly, we calculate the sixth order Box-Ljung (1978) statistic for the

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specifications. Finally, the cross return series specifications for the conditional mean (ϕ_{15}) are standardised residuals (Q(6)) and squared residuals ($Q^2(6)$) from each bivariate estimation for all series and the accompanying market index series. We find no evidence of serial correlation in neither residuals nor squared residuals up to lag 6. Secondly, all the bivariate estimations show no significant cross-correlation at any lags (not reported). Thirdly, the numbers for kurtosis and skews for the standardised residuals report excess kurtosis, but importantly, the numbers for kurtosis and skew are strongly reduced relative to the adjusted raw data series. Our results therefore suggest that the bivariate ARMA-GARCH-in-Mean filter specification produce more normal time series residuals. These results are confirmed by strongly reduced K-S Z-test statistics from Table 1. However, the K-S Z-test statistic still disputes normality for all series.

{Insert Table 3 about here}

Fourthly, the ARCH test statistic reports insignificant test statistics for conditional heteroscedasticity for all series. The RESET test statistic report an insignificant test statistics implying no data-dependence in the conditional mean. Finally the Brock and Deckert (1988) and Scheinkmann (1990) (BDS) test statistic report general non-linear dependence at some dimension (*m*) 2 to 6, for $\varepsilon = 1$, for all thinly traded series, while frequently traded series report no general non-linear dependence.

5 Findings and Characteristics from the Norwegian thinly traded market

The findings from these bivariate ARMA-GARCH-in-Mean estimations may bring some new insights to thinly traded market dynamics. Firstly, the estimations suggest a clear pattern in the "zero-beta" return. The zero-beta return is clearly more significant for frequently traded assets. This result suggests that the drift show a lower daily variance for frequently traded assets than thinly traded assets. Hence, frequently traded assets seem to report a more regular daily positive return.

Secondly, none of the series report significant in-Mean coefficients (λ). Hence, the residual risks specification is rejected, which also suggests rejection of conditional multifactor models. The one dynamic factor hypothesis is rejected and the conditional CAPM specification is rejected. Consequently, all alternative conditional mean series specifications in bivariate ARMA-GARCH-in-Mean lag form do not add extra information to the conditional mean and our specification is not able to distinguish between alternative asset pricing hypotheses. The result suggests that the market show no short-term risk compensations. Moreover, the market index produces insignificant mean coefficients from its own conditional variance process. As all the coefficients are negative for the market, the result suggests lower returns during high volatility regimes, which seem to fit well with observed facts and the volatility-feedback hypothesis (Campbell and Hentschell, 1992).

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determined by a likelihood ratio test among competing specifications.

The autocorrelation results in the conditional means suggest non-synchronous trading and non-trading effects. For frequently traded asset series and the market index our results imply a significant negative MA(1) coefficient. The negative coefficient induces positive autocorrelation, which imply slow adjustment to shocks. However, non-synchronous trading may produce spurious positive autocorrelation[16]. For thinly traded assets our results suggest significant negative autocorrelation. The thinly traded series therefore seem to report overreaction and mean reversion. The results are clearly influenced by non-trading effects and therefore long series of zero returns. Hence, also here the non-synchronous trading effects may induce spurious autocorrelation in the conditional mean.

Conditional heteroscedasticity is present in all series. Moreover, the market index influence in the conditional variance of asset series induces that the variance process for assets may not be modelled as an univariate processes. In our bivariate specifications the series own past conditional variance, the past conditional market index variance and the past conditional covariance significantly explain the volatility of the return series. Hence, the results strongly induce a preference for a bivariate relative to univariate specifications. It is therefore naturally to assume that an univariate representation does not adequately capture all temporal dependencies in the Norwegian equity market. Moreover, the GARCH coefficients also seem to induce that only past shocks and past conditional variance from frequently traded assets significantly influence the conditional variance of the market.

Finally, applying the specification test results induce several interesting findings. Firstly, for all bivariate series the ARCH test reports insignificant statistics. Hence, the result suggests that all conditional heteroscedasticity is removed form the series. For frequently traded assets the RESET and the BDS test statistics report insignificant statistics. Hence, for these bivariate ARMA-GARCH filter residuals neither the independence nor the Martingale hypotheses can be rejected. Hence, the filter implies that the lag specification adequately models the market dynamics for frequently traded assets. In contrast, for thinly traded assets, the RESET test statistics report insignificant values wile the BDS test statistics report significant statistics at some dimension (*m*). Hence, these assets report no conditional heteroscedasticity, no data-dependence in mean but general non-linear dependence. Hence, non-synchronous trading and non-trading suggest data-dependence not possible to model in classical bivariate ARMA-GARCH lag specification models. More elaborate models non-trading models need to be developed, which may apply virtual returns and explicitly counts non-trading periods (Campbell, 1997, Drost and Niemann, 1993)[17]. Hence, our results suggest that non-linearity in frequently stock returns originates from conditional heteroscedasticity, while thinly traded stocks seem to exhibit non-trading effects that a linear ARMA specification of the conditional

[16] For the sub-period 1987-1994 this MA(1) coefficient turns insignificant.

mean cannot adequately model. However, for assets not subject to strong non-trading the bivariate ARMA-GARCH specification seems robust. Note that for thinly traded assets our results suggest that intuitive, analytical and linear reasoning may turn extremely difficult. Economic implications may be even more difficult to interpret.

5.1 Co-variance Characteristics in the Norwegian thinly traded market

The ARMA-GARCH lag specification can be used to create and analyse the conditional variance and covariance matrix. We report the volatility characteristics in Tables 4. The conditional variance means and fluctuations are strongly higher for thinly traded series relative to frequently traded series. The conditional covariance mean seems to be higher for frequently traded series while the fluctuations in the covariance seems to be higher for thinly traded series. These results suggest higher market sensitivity (β) for highly traded series. The mean of the conditional beta measure is highest for the frequently traded series while the standard deviation of the beta is clearly higher for the thinly traded series.

A closer examination of the time varying covariance series also reveals negative covariance in the bivariate estimation for the thinly traded series. Moreover, both the mean and the standard deviation for the conditional covariance seem to increase in ascending order of trading volume portfolios. Hence, the betas should increase as the trading volume increases.

{Insert Table 4 about here}

To see if we find any relation between trading frequency and beta, we study the cumulative frequency distribution of the conditional time varying beta measure. We start by finding the frequency of the $\beta_{i,t-1}$ observations in a interval between -1 and 3 (bin-interval). We move on to accumulate the observations and define an empirical cumulative distribution function of the $\beta_{i,t-1}$ observations for each series. The cumulative distributions are plotted in Figure 1, with dotted lines for individual assets and lines for portfolios. The ordering of both assets and portfolios is of obvious interest. Figure 1 shows a cumulative distribution of the $\beta_{i,t-1}$ observations, that sorts the portfolios nicely in ascending order of trading frequency. Hence, the result seems to imply that the highest relevant risk will be found for the frequently traded series and lowest relevant risk will be found for the thinly traded series. Note especially that applying portfolio theory, the close to zero and negative beta series for the thinly traded assets may be of considerable interest for portfolio managers. Negative betas are usually very desirable in building asset portfolios. However, the specifications tests above suggest that these beta (β) results may originate from serious biases in the co-moments of the return series.

[17] An univariate version of ARMA-GARCH non-trading specification is already established at Molde College (Solibakke, 2000)

{Insert Figure 1 about here}

6 Summaries and Conclusions

This paper has estimated a bivariate ARMA-GARCH-in-Mean specification of the conditional CAPM in the Norwegian thinly traded equity market, controlling for non-synchronous trading and conditional heteroscedasticity. The bivariate conditional CAPM specification captures non-synchronous trading and conditional heteroscedasticity in asset series. Moreover, our specification captures the “leverage” effect (Nelson, 1991) in the bivariate conditional variance equations. The estimations focus on moment and co-moments characteristics in the Norwegian thinly traded market.

The in-Mean specification is redundant as all series report insignificant variance and covariance parameters in the conditional means. As a consequence, the dominance test of the conditional CAPM model versus the residual risk and the one dynamic factor model is left unsettled. Non-synchronous trading and non-trading effects are present in the Norwegian thinly traded market. The thinly traded series report strong mean reversion while frequently traded assets as well as the market index report significant slow adjustment.

The ARCH- and GARCH-coefficients in the bivariate system of equations are for almost all coefficients strongly significant. The results imply firstly, conditional heteroscedasticity and secondly, a univariate specification may not capture enough market dynamics. Specification tests report rejection of thinly traded asset specification while frequently traded assets show adequate model specification. Hence, the data dependence in thinly traded assets induce a wrongly specified model for these assets and suggest a need for more elaborate models for daily return observations in thin markets. Finally, we find that the cumulative frequency distributions of the risk measure β, can be sorted according to an ascending order of trading frequency. The frequently traded assets and portfolio is the most risky measured by the conditional β series. However, the specification tests failures for thinly traded assets induce spurious moments and co-moments characteristics. The moment and co-moments results for thinly traded assets must therefore be treated by considerable scepticism. Consequently, analytical, intuitive and linear reasoning and economic implications become very difficult. The non-trading issue in thinly traded markets must therefore be left to future research.

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