Chapter V

Testing the Univariate Conditional CAPM in Thinly Traded Markets: Controlling for Non-synchronous Trading and Volatility Clustering

Abstract.

Traditional tests of asset pricing undertaken within the CAPM framework have not controlled for non-synchronous trading and non-trading as well as volatility clustering in thinly traded financial markets. This investigation set out to control non-synchronous trading and non-trading effects and volatility clustering in the thinly traded Norwegian equity market. We apply a linear ARMA-GARCH-in-Mean lag specification. The ARMA mean specification control for non-synchronous trading and non-trading, which may causes the observed serial correlation in the mean. The GARCH volatility specification control for conditional heteroscedasticity, which may causes the observed time-variation and volatility clustering in the conditional volatility. Our results suggest that the conditional CAPM cannot be rejected but the in-mean parameter in ARMA-GARCH-in-Mean specifications show very low statistical significance except for perhaps daily data. Our result therefore suggests a compensation for risk only for short time-horizons and that the in-mean parameter in ARMA-GARCH-in-Mean lags specification is a poor proxy for risk in the conditional CAPM sense. Conditional heteroscedasticity and volatility clustering need to be controlled for in daily and weekly time intervals while non-synchronous trading needs to be controlled for daily, weekly and monthly time intervals. Hence, the ARMA-GARCH-in-Mean model is strongly preferred to an unconditional model for daily and weekly sampling intervals. Moreover, for the same intervals the unconditional model shows misspecification, which induces rejection of constant volatility for all series. For monthly sampling intervals volatility clustering is not present in the raw series while non-synchronous trading and non-trading effects seem to persist in the most thinly traded series.

JEL Classification: G12.

Keywords: Conditional CAPM, Non-synchronous trading and non-trading, Volatility clustering, ARMA-GARCH-in-Mean model

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1 Introduction and literature review

From a theoretical perspective, the Capital Asset Pricing Model (CAPM) of Sharpe (1963), Lintner (1965), Mossin (1966) and Black (1972) is a one-period equilibrium model and as such, it is not designed to account for temporal dependence and non-synchronous trading. From a practical perspective, it is well known that the distributions of asset returns exhibit volatility clustering, which manifest itself as temporal dependence in variances, and non-synchronous trading and non-trading effects, which manifest itself as autocorrelation in mean. Turtle (1994) has shown that serial correlation will be induced into model disturbances, when conditional variances are time varying. Campbell et al. (1997) has shown that spurious autocorrelation will be induced into model disturbances. Consequently, tests of an unconditional CAPM in the thinly traded Norwegian market may be wrongly specified.

Researchers have attempted to test the CAPM in a conditional framework utilizing the generalized auto-regressive conditional heteroscedastcity in-mean (GARCH-in-Mean) model (Engle, 1982; Engle and Bollerslev, 1986) in which asset returns are modeled as a function of their conditional variance. Examples include Baillie and DeGennaro (1990), French et al. (1987) and Harris (1989). However, while models that explicitly allow for ARCH effects have been reasonably successful in modeling financial time series, the GARCH-in-Mean model has typically been used as a pure statistical description of returns. Turtle (1994) has provided a theoretical asset pricing application of the GARCH-in-Mean model. Turtle’s results from the US market suggest that while the GARCH-in-Mean model cannot be rejected, the conditional CAPM can be rejected. Moreover, the Brailsford and Faff (1997) results from the Australian market cannot reject the GARCH-in-Mean model in daily and weekly series but can be rejected for monthly series while the conditional CAPM cannot be rejected for weekly and monthly series but can be rejected for daily series.

Hence, the purpose of this paper is to apply and extend Turtle’s (1994) US market results and Brailford and Faff (1997) Australian analyses to the Norwegian thinly traded market. It is of international interest to see how robust these US and Australian equity market results are in the context of thinly traded markets. Thinly traded markets exhibit non-synchronous trading and non-trading effects, which may create negative serial correlation in asset returns (Solibakke, 2000a). To control for the serial correlation we apply an ARMA lag specification for the conditional mean. Moreover, as the GARCH specifications apply lagged residuals in the volatility specification, non-synchronous trading and non-trading may create spurious conditional volatility effects (Solibakke, 2000a). Equally important a second objective for this paper is the effect of sampling interval on the empirical performance of the conditional CAPM and the ARMA-GARCH-in-Mean specification. The objective is especially important for thinly traded markets as market indices may contain assets not traded for days and possibly weeks.

We are therefore able to see how important it is to control for non-synchronous trading over sampling intervals.

We apply the BIC criterion (Schwarz, 1978) to measure the optimal lag structure both in the conditional mean and in the conditional variance equations. We are therefore able to find the optimal lag structure for all series at all sampling intervals. We employ both value-weighted and equal-weighted market index series to see whether the weighting of the market indices influences our findings. The equal-weighted indices will most likely show the highest influence from non-synchronous trading and non-trading effects.

The study differs therefore from other similar studies in especially two ways. Firstly, the use of several index series, equal- and value-weighted contains thinly traded market characteristics. Hence, non-synchronous trading and non-trading effects need to be controlled for and we apply the BIC criterion (Schwarz, 1978) for serial correlation adjustment in the conditional mean equation. Secondly, we apply the BIC criterion (Schwarz, 1978) to find the optimal GARCH specification for conditional heteroscedasticity in the conditional variance equation. Thirdly, we apply specification tests to find appropriate specifications for the Norwegian thinly traded market over daily, weekly and monthly sampling intervals.

The rest of the paper is organized as follows. Section 2 describes the CAPM model, the conditional and the unconditional ARMA-GARCH-in-Mean specifications. Section 3 describes the data and adjustments procedures. Section 4 reports the empirical results. Section 5 reports Norwegian findings and finally Section 6 summarizes and concludes.

2 The CAPM and ARMA-GARCH-in-Mean specifications

2.1 The Static CAPM

The Sharpe-Lintner-Mossin CAPM is a one-period model, which describes how assets are priced in equilibrium in terms of the relationship between relevant risk (beta) and expected return. Specifically, the CAPM states that a positive linear relationship should hold between risk and expected return, viz, $E(R_i) = R_F + \beta_i \cdot \left[E(R_M) - R_F\right]$, where $\beta_i = \frac{\sigma_{i,M}}{\sigma_M^2}$, where $\sigma_{i,m}$ denotes the covariance between asset *i* and the market and σ_M^2 denotes the market variance. Note also that we can write $\beta_i = \frac{\sigma_i \cdot \rho_{i,M}}{\sigma_M}$, where $\rho_{i,m}$ denotes the correlation coefficient between asset *i* and the market and σ^2_i denotes asset *i* variance. In their widely cited study, Fama and French (1992) empirically examine the CAPM given above and find that the estimated value of β_i is close to zero. They interpret the “flat” relation between average return and beta as strong evidence against CAPM. The static CAPM is also tested in the Norwegian thinly traded market. However, Carlsen and Ruth (1991) fail to the reject the null of μ_0 significant different from zero for both univariate and multivariate tests[i]. The result may therefore give evidence both for and against the static CAPM, but it is not necessarily evidence for and against the conditional CAPM. The CAPM was developed within the framework of a hypothetical single-period model economy. The real world, however, is dynamic and hence, expected returns and betas are likely to vary over time. Even when expected returns are linear in betas for every time period based on the information available at that time, the relation between the unconditional expected returns and the unconditional beta could be close to zero[ii]. In the next section we assume that CAPM holds in a conditional sense, i.e., it holds at every point in time, based on whatever information is available at that instant.

2.2 The conditional CAPM.

Assuming hedging motives are not sufficient important following Merton (1980) and hence the CAPM will hold in conditional sense and if we assume that expectations in CAPM at time *t* are conditioned on the information set available to agents at time *t-1*, Ω_{t-1}, then the conditional CAPM[iii] can be written as $E_t(R_{i,t}|\Omega_{t-1}) = R_{F,t-1} + \delta_{1,t-1} \cdot \beta_{i,t-1}$, where $\beta_{i,t-1}$ is the conditional beta of asset *i* defined as $\beta_{i,t-1} = \dfrac{Cov(R_{i,t}, R_{M,t}|\Omega_{t-1})}{Var(R_{M,t}|\Omega_{t-1})}$. $R_{F,t-1}$ is the risk free rate, and $\delta_{1,t-1}$ is the conditional market risk premium defined as $E_t(R_{M,t}|\Omega_{t-1}) - R_{F,t-1}$.

As the model is now stated it is not operational because of the lack of an observed series for the expected market excess return. However the conditional CAPM model assumes neither the beta nor the risk premium is to be constant over time. If we now reformulate the conditional CAPM and write

$E_t(R_{i,t}|\Omega_{t-1}) = R_{F,t-1} + Cov(R_{i,t}, R_{M,t}|\Omega_{t-1}) \cdot \dfrac{\delta_{1,t-1}}{Var(R_{M,t}|\Omega_{t-1})}$, then we have defined the ratio between the conditional risk premium and the conditional variance of the market portfolio. This ratio, defined as the aggregate risk aversion coefficient λ, can be assumed constant over the sample time periods. Therefore, a testable version of the conditional CAPM is given by the specification

$$E_t(R_{i,t}|\Omega_{t-1}) = R_{F,t-1} + Cov(R_{i,t}, R_{M,t}|\Omega_{t-1}) \cdot \lambda_{i,t-1} \qquad (1)$$

Alternatively, model (1) can be expressed as $E_t(r_{i,t}|\Omega_{t-1}) = \lambda_{i,t-1} \cdot Cov_t(t_{i,t}, r_{M,t}|\Omega_{t-1})$, where $\lambda_{i,t-1} = \dfrac{E_t(r_M|\Omega_{t-1})}{Var_t(r_M|\Omega_{t-1})}$, and $E_t(r_i) = E_t(R_i) - R_F$ and $E_t(r_M) = E_t(R_M) - R_F$.

In a multivariate setting this model (1) requires the specification of the dynamics of $Cov(R_{i,t}, R_{M,t} | \Omega_{t-1})$. However, if the model is analysed for the special case where $i = M$, then the model becomes $E_t(r_{M,t}|\Omega_{t-1}) = \lambda_{i,t-1} \cdot Var_t(r_{M,t}|\Omega_{t-1})$, where $\lambda_{i,t-1} = \frac{E_t(r_M|\Omega_{t-1})}{Var_t(r_M|\Omega_{t-1})}$ is assumed constant. This theoretical specification provides the central focus of the tests conducted in this paper.

2.3 The Conditional and unconditional ARMA-GARCH-in-Mean model

The empirical counterpart of the last equation in the previous section is given by

$$r_{M,t} = \mu_{M,t-1} + \sum_{i=1}^{p} \phi_{M,i} \cdot r_{M,t-i} + \lambda_{M,t-1} \cdot \sigma^2_{M,t} - \sum_{j=1}^{q} \theta_{M,j} \cdot \varepsilon_{M,t-j} + \varepsilon_{M,t} \tag{2}$$

where $r_{M,t}$ is excess return on the market and $\sigma^2_{M,t}$ is variance of excess returns on the market both in period t. $\mu_{M,t-1}$ and $\lambda_{M,t-1}$ are constant coefficients for intercept and slope, respectively. This ARMA (p,q) specification and as suggested by Turtle (1994), the model in (1) and empirical counterpart in (2), can be estimated as an univariate ARMA-GARCH-M model process or as a simple conditional mean model (ARMA). Assuming now that an ARMA(p,q)-GARCH (m,n)-in-Mean model is appropriate, (2) is supplemented by the conditional volatility equation given by

$$\sigma^2_{M,t} = m_0 + \sum_{r=1}^{m} a_r \cdot \varepsilon^2_{M,t-r} + \sum_{s=1}^{n} b_s \cdot \sigma^2_{M,t-s} \tag{3}$$

The null hypothesis that the conditional CAPM is 'true' in the context of (1) and (2) is given by H_0: $\mu = 0$. Hence, if $\mu = 0$ in (2) then the excess market return is explained by only its conditional variance consistent with the conditional CAPM. Alternatively, the model can be estimated in its unconditional form:

$$r_{M,t} = E_t(r_M) + v_t \tag{4}$$

where $E_t(r_M)$ is the conditional expectation of the excess return of the market. A comparison of the conditional model in (2) and (3) with the unconditional model in (4), gives rise to a set of subsidiary tests. Specifically, the null hypothesis is given *by* H_0: $\lambda = a_r = b_s = 0$. Hence, if these restrictions are imposed on (2) and (3) the conditional model collapses to its unconditional form given by (4).

Finally, it is interesting to compare the estimates of the unconditional mean and variance of excess market returns implied by the GARCH-M model to their counterparts from the unconditional model. A close similarity between these estimates would suggest that the conditional model is performing well empirically. For example, the unconditional mean of excess market returns implied by the (conditional) ARMA-GARCH-M model is given by

$\hat{\mu} + \hat{\lambda} \cdot \sigma_{M,t}^2 (Volatility)$. Moreover, the unconditional volatility of market excess returns implied by the ARMA-GARCH-M model is given by $\frac{\hat{m}_0}{\left[1 - \left(\sum \hat{a}_r + \sum \hat{b}_s\right)\right]}$.

3 Data and data adjustment procedures

The study uses daily return series from the Norwegian equity market spanning the period from October 1983 to February 1994. We employ four market wide indices. *Pindx* and *Pequl* are indices from Oslo Stock exchange, value-weighted and equal-weighted, respectively. *Nhhvw* and *Nhhew* are two indices from "Børsdataprosjektet" at the Norwegian School of Economics and Business Administration (NHH), value-weighted and equal-weighted respectively. For the two indices from Oslo Stock Exchange the entire 10 years' time period 1983-1994 give a total of 2611 daily observations. The two indices from NHH support daily observations from 1. January 1984; that is, 2525 daily observations. The crash is not excluded from the sample. We therefore assume that crashes are "normal" in equity markets. Moreover, as we need excess returns a proxy for the risk free rate of return is needed. We obtain a 90-day bank accepted bill series from the Central Bank of Norway. Finally, we adjust for systematic location and scale effects (Gallant, Rossi and Tauchen, 1992) in all series.

The log first difference of the price index is adjusted. Let ϖ denote the variable to be adjusted. Initially, the regression to the mean equation $\varpi = x \cdot \beta + u$ is fitted, where *x* consists of calendar variables as are most convenient for the time series and contains parameters for trends, week dummies, calendar day separation variable, month and sub-periods. To the residuals, $\hat{u}$, the variance equation model $\hat{u}^2 = x \cdot \gamma + \varepsilon$ is estimated. Next $\frac{\hat{u}^2}{\sqrt{e^{x \cdot \hat{\gamma}}}}$ is formed, leaving a series with mean zero and (approximately) unit variance given *x*. Lastly, the series $\hat{\varpi} = a + b \cdot (\frac{\hat{u}}{\sqrt{e^{x\hat{\gamma}}}})$ is taken as the adjusted series, where *a* and *b* are chosen so that $\frac{1}{T} \cdot \sum_{i=1}^{T} \hat{\varpi}_i = \frac{1}{T} \cdot \sum_{i=1}^{T} \varpi_i$ and $\frac{1}{T-1} \cdot \sum_{i=1}^{T} (\hat{\varpi}_i - \overline{\varpi})^2 = \frac{1}{T-1} \cdot \sum_{i=1}^{T} (\hat{u}_i - \overline{u})^2$. The purpose of the final location and scale transformation is to aid interpretation. In particular, the unit of measurement of the adjusted series is the same as that of the original series. We do not report the result of these raw data series adjustments[iv]. Characteristics for the excess return of the four indices and three sampling intervals are reported in Table 1.

Following immediate observations can be extracted. The mean and standard deviation shows no clear patterns over sampling intervals and indices. The maximum and minimum returns report remarkable similarities over sampling intervals. The numbers for kurtosis and skew for daily return series suggest a substantial deviation from the normal distribution, which is confirmed by the K-S Z-test. For weekly series the K-S Z-test report close to normally distributed returns for the value-weighted indices, while the equal-weighted indices show deviation from normality. For monthly series normality cannot be rejected. Moreover, the skew is significantly negative for all indices and all sampling intervals. The kurtosis and skews together, suggest too much probability mass around the mean, too little around 1-2 standard deviation from the mean and more extreme values on especially the negative side of the distributions. The results suggest a need for heavy tails specification for especially daily but also weekly returns. We therefore employ a student-t distribution (Blattberg and Gonedes, 1974) for daily and weekly returns while monthly returns employ normal distributions[v].

{INSERT TABLE 1 ABOUT HERE}

The Ljung and Box statistics Q(6) and $Q^2(6)$, reports evidence of serial correlation up to lag 6 for return and squared return series, respectively, for daily and weekly sampling intervals. For monthly series the value-weighted indices report no serial correlation, while the equal-weighted indices report serial correlation. These results suggest non-synchronous trading and non-trading effects in almost all indices. Hence, the series need a linear ARMA lag specifications for the conditional mean in model (2) and (4). Gallant and Tauchen (1997) and Solibakke (2000) among others, employ the BIC criterion (Schwarz, 1978) for optimal lag structures in ARMA specifications. An ARMA (0,1) model is chosen for the two value-weighted daily indices and an ARMA (1,0) for the equal-weighted daily indices. For weekly {monthly} returns we obtain ARMA (1,0) {ARMA (0,1)} for the value-weighted and ARMA (2,0) {ARMA (1,0)} for equal-weighted indices.

The $Q^2(6)$ and the ARCH test (Engle, 1982) report serial correlation and autoregressive and conditional heteroscedasticity (ARCH) in the squared return series for all series and sampling intervals except monthly intervals. Hence, volatility clustering and changing volatility is found in all daily and weekly series. Applying the BIC criterion (Schwarz, 1978) on the squared residuals from the above defined ARMA lag specifications, all indices and sampling intervals prefers a GARCH (1,1) specification[vi]. However, monthly series may show insignificant GARCH parameters.

Finally, the RESET (Ramsey, 1969) and the BDS (Brock and Deckert, 1988 and Scheinkman, 1990) for $m = 2, 3$ and 4 and $\varepsilon = 1$ specification test statistics both report data dependence in all adjusted raw data series. The RESET test suggests data dependence in the mean and the BDS test suggests general nonlinear dependence in all series at some dimension (m). The BDS test statistic reports a surprisingly stable and strongly significant nonlinear dependence in all market indices for all sampling intervals. Hence, we employ these test statistics to measure data-dependence after applying ARMA-GARCH filters for all series. Any significant test values induce specification errors in our BIC preferred lag model and will probably make economic implications difficult to interpret.

4 Empirical Results

The results of estimating[vii] the ARMA-GARCH-in-Mean model for excess market return series are presented in Table 2 for all four indices and all three sampling intervals. For each series and sampling intervals the first two lines report an unrestricted version of (2) while line three and four report a restricted version ($\mu = 0$), which is consistent with the conditional CAPM.

The most notable result across all measurement intervals and indices is the general lack of significance of the in-mean parameter. This appears to indicate that the GARCH-in-Mean model is an inadequate mean of capturing asset-pricing dynamics. However, this finding is somewhat offset by the insignificant coefficient estimates on μ in the unrestricted models for all return series. Hence, firstly, consider the first and second row for all indices and sampling intervals in Table 2. The first and second rows present the coefficient estimates and the t-statistics (in brackets), respectively, for the unrestricted model. All coefficient estimates for the in-mean parameter (λ) and the constant parameter (μ) are insignificant. The insignificant coefficient estimate of μ is indicative of acceptance of the conditional CAPM.

Secondly, consider the third and fourth row for all series and sampling intervals. The third and fourth rows present the coefficient estimates and the t-statistics (in brackets), respectively, for the restricted model *($\mu = 0$)*. Only for daily return series the estimation produce a positive and significant in-mean parameter (λ). It is this restricted version of the GARCH-in-Mean model, which is implied by the conditional CAPM. The implication of this result is that investors are compensated for risk only over very short time horizons (i.e. daily).

Thirdly, for monthly return series the coefficient estimates show overall low significance. This result is consistent with international research (Bollerslev et al., 1992). The monthly results suggest that the ARMA-GARCH-in-Mean specification for the conditional CAPM should be interpreted with cautions. The ARMA-GARCH-in-Mean model specification performs best for shorter time horizons, as suggested by several of the test statistics in Table 1.

Fourthly, the student-t distribution parameter υ (degrees of freedom) is for all estimations lower than 7.5 and is therefore strongly significant for daily and weekly return series. The deviation from the normal distribution we found in Table 1 seems therefore to be confirmed in Table 2.

{INSERT TABLE 2 ABOUT HERE}

Table 3 reports the unconditional mean and variance implied by the ARMA-GARCH-in-Mean model in (2) and (3), where λ, a_1 and b_1 is restricted to zero, but μ and φ/θ are free to vary, in contrast to the mean and variance from the unconditional model of market excess return in (4) and (3). In general, from these comparisons it appears that the conditional volatility models provide reasonable estimates of the unconditional variance for all sampling intervals. However, although most likely not significant (suggested by the ARMA-GARCH-M estimation), the conditional mean show considerably lower absolute values for weekly and monthly estimations. For the daily series the unconditional volatility model seems to provide reasonable estimates of the conditional mean. Consequently, the result suggests that the ARMA-GARCH-in-Mean model perform best for the shortest time intervals.

{INSERT TABLE 3 ABOUT HERE}

Table 4, panel A reports the likelihood ratio test of the unrestricted ARMA-GARCH-M model given by (2) and (3) against the conditional CAPM which is implied by the restriction of $\mu = 0$ in (2). The conditional CAPM cannot be rejected for any return series and all sampling intervals at 1%. However, for the value-weighted indices and weekly return intervals the conditional CAPM can be rejected at 5%. Moreover, the acceptance of the CAPM for especially the weekly and monthly return series need to be qualified by the fact that the in-mean parameter estimates were found to be insignificant in line 3 and 4 in Table 2.

Table 4, panel B reports the likelihood ratio tests of the unrestricted ARMA-GARCH-M model given by (2) and (3) against the unconditional model which is implied by the restrictions of $\lambda = a_1 = b_1 = 0$ in (4) and (3). We cannot reject the ARMA-GARCH-M specification for daily and weekly return series, while monthly series reject the ARMA-GARCH-M model specification.

{INSERT TABLE 4 ABOUT HERE}

Generally, the findings are consistent with the use of ARMA-GARCH-M lagged processes to capture the dynamics of asset returns. The processes show improved fits at especially high frequencies. However, our results suggest caution in using such processes as part of formal asset pricing models.

Finally, the diagnostics tests are reported in Table 5 for daily and weekly series[1]. The kurtosis, skew and K-S Z-test statistic report closer to normal standardized residuals. We find no serial correlation (Q(6)) in all series for both the unrestricted and restricted GARCH-M model specifications. For the unconditional model we find significant serial correlation. The reported pattern for serial correlation is also found for the Q^2, the ARCH, the RESET and the BDS test statistics. The test statistics report data dependence for the two equal weighted indices for

[1] Monthly series report for all test statistics insignificant statistics for all three models.

daily sampling intervals. Hence, we cannot reject the ARMA-GARCH-in-Mean specification for the value-weighted index series. However, for the equal weighted index we reject the model for daily series.

{INSERT TABLE 5 ABOUT HERE}

5 Finding from the thinly traded Norwegian market

This investigation performs a test of the conditional CAPM specification. Our main finding is that the conditional CAPM cannot be rejected. However, as indicated by the restricted ARMA-GARCH-in-Mean, which is implied by the conditional CAPM, investors are compensated for risk over the shortest time interval, i.e. daily. Such a conclusion is difficult to accept, as theory would propose that compensation for risk should occur irrespective of the return interval. A possible explanation is that the conditional variance proxy for risk in a generic sense such that it captures liquidity risk, data and measurement error and bid-ask bounce, all of which are greater at shorter return intervals. Hence, the rather crude measure of risk in the model captures market imperfections, which exert their greatest influence at the shortest return interval (daily).

Overall the results suggest that non-synchronous trading and non-trading effects as well as volatility clustering are high at daily series. The effects are strong in both the equal-weighted and the value-weighted indices. For weekly series we still find strong volatility clustering while non-synchronous trading and non-trading effects have decreased strongly for especially the value-weighted indices. In fact, one of the value-weighted indices (*Pindx*) reports insignificant serial correlation in the conditional mean. The equal-weighted indices still report serial correlation. Hence, an interpretation suggests that very thinly traded assets influence the aggregated returns at the weekly sampling interval. Moreover, for non-synchronous trading and non-trading effects monthly series exhibits the same patterns as weekly series. For the value-weighted index series we find insignificant serial correlation while the equal-weighted indices still report significant serial correlation. Hence, non-synchronous trading and non-trading effects are present in the equal-weighted index series also for monthly sampling intervals. However, volatility clustering and changing volatility is not present in monthly series as suggested in Table 1.

Finally, the ARMA-GARCH-in-Mean specification seems to be the preferred model specification for shorter time intervals. For monthly intervals non-synchronous trading is low and volatility clustering is no longer present in the series. Note however, that non-synchronous trading and non-trading effects seem to be present for all sampling intervals for the equal-weighted indices while the value-weighted indices only report daily and weekly effects. Hence, The ARMA-GARCH-in-Mean specification is rejected for the equal weighted indices for daily sampling intervals due to non-trading, while the unconditional model is rejected due to volatility clustering in daily and weekly series. The ARMA-GARCH-in-Mean model for the value-weighted index series rejects misspecification. Non-synchronous trading and non-trading as well as conditional heteroscedasticity and volatility clustering is therefore satisfactory modeled for these index series. More elaborate models need to be developed to account for non-synchronous trading and volatility clustering in high frequency (daily) equal-weighted indices[viii]. Finally, we fail to reject specification errors for all the unconditional volatility models for monthly intervals. Unconditional models are therefore preferred to conditional models for these return intervals.

6 Summaries and conclusions

We have investigated and tested a conditional CAPM in the Norwegian thinly traded equity market. By applying ARMA-GARCH-M lag specifications we control for non-synchronous trading and conditional heteroscedasticity in the dynamics of the market. All lag specifications in the conditional mean and volatility is BIC (Schwarz, 1978) preferred.

We find two factors, which oppose the ARMA-GARCH specification differently. Firstly, monthly series reject conditional heteroscedasticity. Hence, an unconditional model is just as valid as a conditional model. The GARCH volatility specification is therefore superfluous for monthly returns for all series. Secondly, series strongly influenced from non-trading reject the ARMA-GARCH specification for daily series but not weekly and monthly series. Consequently, only the value weighted index series cannot reject the ARMA-GARCH-M specifications for daily and weekly series. The equal-weighted index series cannot reject the ARMA-GARCH-M specifications for weekly series.

The inability of the in-Mean parameter to achieve statistical significance is an empirical limitation for especially weekly and monthly data. Despite these results, we cannot reject the conditional CAPM for any return series intervals. However, the result for the monthly series lacks power as we reject the GARCH model for all monthly series.

In contrast to the unconditional model specification, we find no serial correlation, strongly reduced kurtosis and skew and insignificant ARCH and BDS-test statistic (i.i.d.) for the ARMA-GARCH-in-Mean specification daily value-weighted index series and weekly value-weighted and equal-weighed index series. Hence, the ARMA-GARCH-in-Mean model reports no model misspecification for weekly and monthly intervals. This result implies that severe non-synchronous trading and non-trading induces ARMA-GARCH misspecification.

In summary, the ARMA-GARCH-M model is a useful empirical tool for modeling equity return series at high frequencies such as daily and weekly return intervals for thinly traded markets. However, severe non-synchronous trading and non-trading may cause ARMA-GARCH misspecification for high frequency series. Moreover, some care must be taken in placing economic significance to the model in an asset-pricing regime. As the return interval increases, the superiority of the model decreases, consistent with prior international empirical and theoretical work. For the Norwegian market only monthly series reject the CAPM specification. Moreover, within model validity intervals, the return interval appears not to influence the insignificant rejection result for the conditional CAPM. However, for weekly intervals and the value-weighted indices the conditional CAPM is rejected at 5%. We may therefore induce that for thinly traded markets and for both daily and weekly return interval the ARMA-GARCH-in-Mean specifications are appropriate for asset tests. However, the problems of modeling virtually non-existing time-varying dynamics for the monthly interval in these markets make monthly return intervals probably not appropriate for such tests. Finally, thinly traded markets need more elaborate asset models for high frequency (daily) market dynamics, as indicated by specification test rejections of the ARMA-GARCH-in-Mean model.

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[i] See also Carlsen and Ruth, 1990, Stange , 1989, Semmen, 1989 and Hatlen et al. 1988.

[ii] Because an asset that is on the conditional mean-variance frontier need not be on the unconditional frontier (Dybvig and Ross, 1985 and Hansen and Richard, 1987)

[iii] See Jagannathan and Wang, 1996.

[iv] The results are readily available from the author upon request.

[v] See Berndt et al., 1974, for a detailed description of the iterative optimization routines (BHHH).

[vi] This result conform to other international findings and is documented for the Norwegian market for daily returns in Solibakke, 1999a.

[vii] We employ the BHHH algorithm of Berndt et al. (1974).

[viii] Solibakke (2000) employ virtual returns and continous time ARMA-GARCH lag specifications to model non-synchronous trading and conditional heteroscedasticity.