Chapter IV

Nonlinear Dependence in Thinly Traded Markets

Abstract.

We investigate the presence of nonlinear dependencies in stock returns for the Norwegian thinly traded equity market. It is well known that it is very difficult to interpret the unconditional distribution of stock returns and its economic implications if the i.i.d. assumption is violated. We employ ARMA-GARCH lag specifications for the conditional mean and volatility processes modeling non-synchronous trading and volatility clustering characteristics. Any presence of nonlinear dependence must distinguish between models that are non-linear in mean and hence depart from the Martingale hypothesis, and models that are nonlinear in volatility and hence depart from independence but not from the Martingale hypothesis. Our investigation start by answering which model that seems to have the necessary characteristics to account for the nonlinear dependence in the Norwegian market. Our results suggest that the observed nonlinear dependence seems to be conditional heteroscedasticity and volatility clustering. Hence, most of the nonlinear dependence in adjusted raw returns is found in the conditional volatility process. However, thinly traded assets report significant nonlinear dependence for all model specifications. Consequently, the ARMA-GARCH model specification seems not appropriate for thinly traded series. We reject the independence hypothesis but fail to reject the Martingale hypothesis for any series.

Classification:

Keywords: Non-synchronous-trading and non-trading, volatility clustering, data-dependence, independence, Martingales

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1 Introduction

Nonlinear dependence in stock returns has recently attracted much attention. Examples are Abhyankar et al. (1995) from the UK market and de Lima (1995 a, b), Hsieh (1991) Brock et al. (1991), and Lee et al. (1993) from the US market[i]. Nonlinear structure in univariate time series departs from the random walk model and will be unfamiliar territory to those who are accustomed to thinking analytically, intuitively, and linearly. The Random Walk model (Bachelier, 1964), which assumes that security prices from transaction to transaction are independent, identically distributed (i.i.d) random variables, together with the central limit theorem, suggests that price changes are normally distributed and that their variances will be linearly related to the time interval. Moreover, as noted by Hsieh (1991), it is difficult to interpret the unconditional distribution of stock returns and its economic implications, if the i.i.d. assumption is violated. If stock returns are i.i.d. and follow fat tails distributions such as Cauchy (Mandelbrot, 1963), the Student-t density (Blattberg and Gonedes, 1974) or Normal Inverse Gaussian (Eberlein and Keller, 1994 and Barndorff-Nielsen, 1994) the probability of observing large absolute returns such as that on 19-22 October 1987 is small but non-zero. In this case market crashes such as that of the 1987 could happen at any time but with very low probability (Brown, Goetzman and Ross, 1995). The behavior of risk adverse agents will consequently take this into account (Bollerslev et al., 1993). Our crucial point is that such an interpretation is so dependent on the i.i.d. assumption since the unconditional distribution will always have fatter tails than the conditional distribution if the data has some form of conditional dependence[ii].

One prominent explanation for the observed departure from Bachelier's (1964) model is the mixture of distributions hypothesis (Epps and Epps, 1976 and Tauchen and Pitts, 1983). This maintains that trade-to-trade asset returns exhibit leptokurtosis because they are really a combination of return distributions that are conditioned on information arrival. This means that periods of little or no information arrival result in different observed return distributions than in periods when information frequently arrive (Clark, 1973, Harris, 1989 and French and Roll, 1986). Hence, the characteristics of assets that exhibit low information flow may depart from assets that exhibit high information flow.

Another important departure from the Random Walk model is found in the microstructure literature. This literature describes a trading process that exhibits non-synchronous trading and non-trading effects, which arises when return series are taken to be recorded at time intervals of one length when in fact they are recorded at time intervals of other, possible irregular, lengths. For example, the daily closing prices of the Norwegian firm Farstad Shipping (Aalesund) are quoted on the Oslo Stock Exchange and reported daily in Dagens Næringsliv. Note that the closing price reported in Dagens Næringsliv is the price at which the last transaction in Farstad Shipping occurred on the previous day. In a thinly traded equity market the closing price will generally not occur at the same time each day. Hence, Farstad Shipping may on one particular Monday quote its last reported trade at $14^{\underline{05}}$, which will become the closing price reported in Dagens Næringsliv that particular Monday even though the Oslo Stock Exchange closes at $16^{\underline{00}}$. Moreover, the following day Tuesday, the last quoted trade was reported at $15^{\underline{15}}$. This example shows that referring to them as “daily” prices, we have implicitly and incorrectly assumed that they are equally spaced in 24-hour intervals. Moreover, Farstad Shipping reported zero trading volume for several days in 1999, that is, several days of non-trading. Non-synchronous trading and nontrading may induce potentially biases in the moments and co-moments of assets returns. The Norwegian equity market exhibits low trading volume relative to elaborate markets in US and UK and contains assets that show low trading volume relative to continuously traded assets (solibakke, 2000). The market may therefore exhibit strong non-synchronous trading and non-trading effects. In the same vein as for the information flow, the characteristics of thinly traded assets may not be the same as that for actively traded assets (Gallant, Rossi and Tauchen, 1992).

To investigate non-synchronous trading and non-trading effects for the Norwegian thinly traded equity market, we investigate nonlinear dependence and departure from the i.i.d. assumption in return series. We employ seven individual assets, two trading volume portfolios and an equal-weighted and a value weighted index. Results in Solibakke (2000) suggest that a null hypothesis of i.i.d. for ARMA-GARCH filtered adjusted return series may be rejected. The main objective for this investigation is therefore to find any systematic difference in nonlinear dependence over a wide trading volume range (including non-trading) and whether non-synchronous trading and non-trading effects may characterize nonlinear dependence. The nonlinear dependence in univariate time series applying ARMA-GARCH lag structures may be specified by (1) nonlinear dependence in mean, (2) nonlinear dependence in variance or (3) nonlinear dependence in both mean and variance. Moreover, note that modeling of nonlinear dependence must distinguish between models that are non-linear in mean and hence depart from the Martingale hypothesis, and models that are nonlinear in variance and hence depart from the assumption of independence, but not from the Martingale hypothesis.

Our nonlinear dependence investigation examines three different non-linear conditional mean and volatility ARMA-GARCH specifications, attempting to filter out the observed nonlinear dependence in the Norwegian return series. We employ a non-synchronous trading (ARMA) lag specification for the conditional mean and a conditional heteroscedasticity (ARCH/GARCH) lag specification for the conditional volatility. Observed characteristics as leptokurtosis and asymmetric volatility are incorporated into the model specifications, applying normal and student-t density log-likelihood functions and the GJR-GARCH specification of Glosten et al. (1993). Hence, we hypothesize nonlinear dependence in the Norwegian equity market employing these specifications for the conditional mean and volatility equations for all return series.

The study differs from other studies (Abhyankar et al., 1995) in several ways. Firstly, we apply three elaborate test statistics for nonlinear dependence. The test statistics distinguish between non-linearity in the conditional mean and volatility. Secondly, we apply thinly and frequently traded asset series and investigate whether individual and aggregate return series behave similar to market index series. Otherwise, any generalization of the findings from aggregate to individual series would be inaccurate. Moreover, applying individual asset series, we may now explicitly study non-synchronous trading and non-trading effects on nonlinear dependence. Thirdly, we employ a normal and a student-t density log-likelihood function for all series. Student-t density functions may account for observed leptokurtosis in stock market returns. Fourthly, we apply the recent adjustment suggested by de Lima (1995 b) to the residuals from the GARCH model before conducting the BDS test statistic (Brock (1988), Dechert (1991) and Scheinkman, 1990). Fifthly, all raw return series are adjusted for systematic size and location effects as suggested by Gallant, Rossi and Tauchen (1992). Sixthly, and finally, the “leverage effect” are modeled in the conditional volatility equations (Nelson, 1991)[iii] applying the GJR-GARCH specification (Glosten et al., 1993).

The rest of the paper is organized as follows. Section 2 specifies three non-linear ARMA-GARCH lag specifications and describes the ARCH (Engle, 1982 and Engle and Bollerslev, 1986), RESET (Ramsey, 1969) and BDS (Brock (1988), Deckert (1991) and Scheinkman (1990)) test statistics for identifying nonlinear dependence. Section 3 describes the Norwegian data and the Gallant, Rossi and Tauchen (1992) adjustment procedures. Section 4 reports the empirical results. Section 5 reports findings and finally Section 6 summarizes and concludes.

2 Specifications of nonlinear relationships and test statistics

2.1 Nonlinear ARMA-GARCH specifications

Many aspects of economic behavior may not be linear. Most evidence and introspection suggest that investor's attitude towards expected return and risks are nonlinear. Moreover, most derivative securities provide nonlinear terms and the strategic interaction between market participants, the process by which information is incorporated into security prices and the dynamics of economy-wide fluctuations are all inherently nonlinear. However, no economic theory or behavior has so far distinguished between nonlinear dependence in conditional mean and variance. Therefore, we have to distinguish between models that are nonlinear in mean and hence depart from the Martingale hypothesis and models that are nonlinear in variance and hence depart from the assumption of independence but not from the Martingale hypothesis.

In nonlinear time-series analysis the underlying shocks are typically assumed to be i.i.d. However, we typically seek a possibly nonlinear function relating the series x_t to the history of shocks. A general representation is $x_t = f(\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}, ...)$ where the shocks are assumed to have mean zero and unit variance, and *f()* is some unknown function. The generality of the representation makes it very hard to work with—most models used in practice fall into a somewhat more restricted class that can be written as $x_t = g(\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}, ...) + \varepsilon_t \cdot h(\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}, ...)$. Here the function *g()* represents the mean of x_t conditional on past information, since $E_{t-1}\left[x_t\right] = g(\varepsilon_{t-1}, \varepsilon_{t-2}, \cdots)$. The innovation in x_t is proportional to the shock ε_t, where the coefficients of proportionality is the function *h()*. The square of this function is the variance of x_t conditional on past information, since $E_{t-1}\left[(x_t - E_{t-1}\left[x_t\right])^2\right] = h(\varepsilon_{t-1}, \varepsilon_{t-2}, \cdots)^2$. Models with nonlinear *g()* are said to be nonlinear in mean, whereas models with nonlinear $h()^2$ are said to be nonlinear in variance. The second equation leads to a natural division in the nonlinear time-series literature between models of the conditional mean *g()* and models of the conditional variance *h()*. Most time-series models concentrate on one form of the non-linearity or the other. However, the (General) Auto-regressive Conditional Heteroscedasticity ((G)ARCH) model of Engle (1982) makes modeling of nonlinear dependence in both mean and variance possible.

Three nonlinear models will be analyzed in this study. A linear ARMA model with a constant (drift) takes the form

$$x_t = \alpha_0 + \sum_{j=1}^{p} \varphi_j \cdot x_{t-j} + \varepsilon_t + \sum_{i=1}^{q} \theta_i \cdot \varepsilon_{t-i} \ ; \qquad h_t = a_0 \tag{1}$$

where *p* and *q* are the BIC (Schwarz, 1978) preferred respective lag lengths; ϕ_i is the auto-regressive parameters and θ_i is the moving average parameters and a_0 is an estimated constant[iv]. The first nonlinear model is an extended ARMA model where non-linearity in the mean is introduced through the squared residual (ε_t^2). This simple non-linear ARMA model takes the form

$$x_t = \alpha_0 + \sum_{j=1}^{p} \varphi_j \cdot x_{t-j} + \varepsilon_t + \sum_{i=1}^{q} \theta_i \cdot \varepsilon_{t-i} + \gamma \cdot \varepsilon_{t-1}^2 \ ; \qquad h_t = a_0 \tag{2}$$

where *p* and *q* is based on the Schwarz Bayesian Criterion (BIC) (1978) from the original adjusted return time-series and a_0 is an estimated constant[v]. The second model analyzes changing volatility and model nonlinear dependence in only the variance equation (the mean is linear). The model takes the form

$$x_t = \alpha_0 + \sum_{j=1}^{p} \varphi_j \cdot x_{t-j} + \varepsilon_t + \sum_{i=1}^{q} \theta_i \cdot \varepsilon_{t-i} \ ; \ \lambda_{it} = \delta_i \quad \text{iff} \quad \varepsilon_{t-i} < 0 \quad \text{and}$$

$$h_t = a_0 + \sum_{j=1}^{m} (a_j + \lambda_{it}) \cdot \varepsilon_{t-i}^2 + \sum_{i=1}^{n} b_i \cdot h_{t-i} \tag{3}$$

where *p* and *q* is based on the BIC Criterion (Schwarz, 1978) from the original data series. By analogy with ARMA models, the third equation in (3) is called a GARCH (*m,n*) model. The coefficient b_i measures the extent to which volatility today feeds through into next period's volatility, while $(\sum a_j + \sum b_i)$ measures the rate at which this effect dies out over time. The GARCH (m,n) model is an ARMA (u,m) model for squared innovations, where $u=max(m,n)$. Therefore, using the BIC criterion for the squared innovation from an ARMA (u,m) model produces the necessary m and n lags. λ_{it} is the vector of parameters for the asymmetric process (leverage)[vi]. Finally, the fourth model combines model (2) and model (3) and takes the form

$$x_t = \alpha_0 + \sum_{j=1}^{p} \varphi_j \cdot x_{t-j} + \varepsilon_t + \sum_{i=1}^{q} \theta_i \cdot \varepsilon_{t-i} + \gamma_1 \cdot \varepsilon_{t-1}^2 + \gamma_2 \cdot h_t \ ;$$

$$\lambda_{it} = \delta_i \quad \text{iff} \quad \varepsilon_{t-i} < 0 \ \text{ and } \ h_t = a_0 + \sum_{j=1}^{m} (a_j + \lambda_{it}) \cdot \varepsilon_{t-i}^2 + \sum_{i=1}^{n} b_i \cdot h_{t-i} \qquad (4)$$

where p, q is based on the BIC criterion (Schwarz, 1978) from the raw data series and m and n is based on the BIC criterion from the squared residuals in a linear ARMA[vii] model. Model (4) is a specification for non-linearity in both conditional mean and conditional variance. All three non-linear models may be estimated under either a normal-[viii] or a student-t[ix] density log-likelihood function.

2.2 Measuring non-linear dependence

2.2.1 The ARCH test statistic

The ARCH test statistic (Engle, 1982) is a test for constant conditional variance against conditional heteroscedasticity, based on the Lagrange Multiplier principle. The test procedure is to run a regression of the squared residuals on a constant and p lagged squared residuals. Then test the test statistic $T \cdot R^2$ as a $\chi^2(p)$ variate, where T is the sample size and R^2 is the squared multiple correlation coefficient and p is the degree of freedom. The ARCH test is a test for H_0: constant conditional variance against the alternative H_a: a conditional variance that obey an ARCH(p) specification. In fact, if ARCH is present in the residuals, nonlinear dependence in the time series cannot be rejected.

2.2.2 The RESET test statistic

The Regression Error Specification Test (RESET; Ramsey, 1969) is a test statistic of linearity against an unspecified alternative. It is a test against general model misspecification[x] and has certainly been one of the most popular tests against misspecification of functional form.
In this paper it is carried out in three stages as follows:

(1) We assume the linear part of the model is

$$y_t = \beta' \cdot z_t + u_t \,, \qquad t = 1,\ldots,T$$

where $z_t = (1, y_{t-1},\ldots y_{t-p}, x_{t1},\ldots,x_{tk})'$. We estimate β by OLS and compute $\hat{u}_t = y_t - \hat{y}_t$ where $\hat{y}_t = \hat{\beta}' \cdot z_t$, and $SSR_0 = \sum \hat{u}_t^2$.

(2) Then we estimate the parameters of $\hat{u}_t = \delta' \cdot z_t + \sum_{j=2}^{h} \varphi_j \tilde{z}_t^{(j)} + \upsilon_t$

by OLS and compute $SSR = \sum \hat{\upsilon}_t^2$, where $\tilde{z}_t^{(j)} = (y_{t-1}^j,\ldots,y_{t-p}^j, x_{t1}^j,\ldots,x_{tk}^j)$, $j = 2,\ldots,h$.

(3) Finally, we compute the test statistic: $F = \dfrac{(SSR_0 - SSR)/(h-1)}{SSR/(T-m-h)}$

where $m = p+k$. k is in our case zero. As z_t contains lags of y_t, then $(h-1)F$ has an asymptotic χ^2 distribution under the null of linearity. h was suggested by Thursby and Schmidt (1977) to 4 for the best result. This test is an Lagrange Multiplier (LM) type test against an Logistic Smooth Transition Regression (LSTR) model in which only one 'linear parameter' changes but the investigator does not know which one. The RESET test is thus rather narrow in that if more than one variable has a 'changing linear parameter' the regression no longer covers that possibility. Note, however, that the constant in the first regression should not be involved in defining the z_t and $\tilde{z}_t$ in the auxiliary regression, since the inclusion of such regressors would lead to perfect collinearity.

2.2.3 The BDS test statistic

2.2.3.1 The correlation integral

The correlation integral proposed by Grassberger and Procaccia (1983) is a measure of spatial correlation in an m-dimensional space. Let $\{\mu_t\}$ be a real-valued scalar time-series process. Construct the m-history process $\mu_t^m \overset{def}{=} (\mu_t, \mu_{t+1}, \cdots, \mu_{t+m-1})$. For $\varepsilon > 0$, the correlation integral at *embedding dimension m* is given by[xi]

$C_{m,\varepsilon} = \iint \chi\varepsilon(x^m, y^m) dF(x^m) dF(y^m)$, where $\chi \cdot \varepsilon(\cdot,\cdot)$ is the symmetric indicator kernel with $\chi \cdot \varepsilon(x, y) = 1$ if $\|x - y\| < \varepsilon$ and 0 otherwise (indicator function), $\|\cdot\|$ represents the max-norm, and $F()$ is the distribution function of μ_t^m. $C_{m,\varepsilon}$ gives the mean volume of a cube with diameter ε. An estimator of the correlation integral for a sample size T for the process $\{\mu_t\}$ is given by the following U-statistic— cf. BDS (1987), $C_{m,\varepsilon} = \dfrac{1}{\binom{\overline{T}}{2}} \sum \sum_{1 \le s < t \le \overline{T}} \chi \cdot \varepsilon(\mu_t^m, \mu_s^m)$,

where $\overline{T} = T - (m-1)$.

2.2.3.2 The test statistic

Brock et al. (1988), Deckert (1991) and Scheinkman (1990), henceforth BDS (Brock, Dechert and Scheinkman), developed a test based on concepts that arise in the theory of chaotic processes. The BDS test statistic is a test of the null hypothesis of i.i.d. for a univariate time series against an unspecified alternative. That is, if $\{\mu_t\}$ is an i.i.d. process, then $C_{m,\varepsilon} = C_{1,\varepsilon}$, almost surely, for all $\varepsilon > 0,\ m = 1, 2, \ldots$. The BDS test presents the following result

$V_{m,\varepsilon} = \sqrt{T} \cdot \dfrac{C_{m,\varepsilon} - (C_{1,\varepsilon})^m}{s_{m,\varepsilon}} \xrightarrow{d} N(0,1)$, $\forall\, \varepsilon > 0,\ m=2,3,\ldots$, where $s_{m,\varepsilon}$ is an estimator of the asymptotic standard deviation—$\sigma_{m,\varepsilon}$—of $\sqrt{T} \cdot (C_{m,\varepsilon} - (C_{1,\varepsilon})^m)$ under the null of i.i.d. Brock et al. (1991) used Monte Carlo methods to evaluate the choice of m and ε on the asymptotic normality of $V_{m,\varepsilon}$. Their results suggest that asymptotic normality of $V_{m,\varepsilon}$ holds well for sample sizes of at least 1000 observations, and for value of ε between 0.5 and 2. They warned against relying on asymptotic normality for values of T/m less than 200 observations.

The BDS test has been shown to be robust to the non-existence of fourth moments, which may characterize stock returns (Brock and de Lima, 1995 and Hsieh, 1991). Hsieh (1991) points out that the robustness of the BDS test to the nonexistence of fourth moments is one of the advantages of the BDS test over other tests of non-linearity such as Tsay (1986) and Hinich and Patterson (1985). Moreover, the BDS test statistic has power against models that are nonlinear in variance but not in mean, as well as models that are nonlinear only in mean. That is, a BDS rejection does not necessarily mean that a time-series has a time-varying conditional mean; it could simply be evidence for a time-varying conditional variance (Hsieh, 1991).

One-way to test whether conditional heteroscedasticity is responsible for the rejection of i.i.d. hypothesis is to apply the BDS test statistic to the residuals from a ARMA - GARCH model (Brock et al 1991, and Abhyankar et al. 1995). The trouble is that we cannot depend on asymptotic normality of the BDS statistic. Hsieh (1991) overcomes this problem by using critical values of the BDS statistic for simulated EGARCH process[xii]. However, a recent paper by de Lima (1995 b) shows that the asymptotic distribution of the BDS statistic remains valid if the test is applied to the natural logarithm of the squared standardized residuals from a GARCH model. This is because the BDS statistic is valid if it is applied to a data generating process that is additive in the error term (de Lima, 1995b). The GARCH process models the error term in a multiplicative form, $\mu_t = \sigma_t \cdot z_t$, where μ_t is a random variable following the GARCH process, z_t is i.i.d. random variable, and σ_t is the conditional standard deviation. The standardized residuals from this model are $z_t = \mu_t / \sigma_t$ in the normal case and

$z_t = \mu_t / \sqrt{\sigma_t^2 \cdot \left(\frac{\eta - 2}{\eta}\right)}$ in the student-t[xiii] density case, where η is the degree of freedom parameter. It follows that $ln(z^2_t) = ln(\mu^2_t) - ln(\sigma^2_t)$ in the normal case and $ln(z^2_t) = ln(\mu^2_t) - ln(\sigma^2_t(\eta\text{-}2/\eta))$ in the student-t density case. Therefore, the asymptotic distribution of the BDS statistic remains valid if it is applied to $ln(z^2_t)$ (adjusted residuals) in both the normal and student-t density case.

3 Data and Adjustment Procedures

The study uses daily returns of individual Norwegian stocks spanning the period from October 1983 to February 1994. The assets examined are assets in the Norwegian equity market. The assets are sorted from frequently traded (no. 1) to thinly traded assets (no. 7). Trading volume is the amount traded of the asset in NOK; that is, the number of stocks traded multiplied by settlement price at time of trading. Moreover, individual shares are grouped into portfolios at period *t* based on trading volume at *t-1*. Portfolio 1 consists of the thinnest traded assets, and portfolio 4 consists of the most frequently (continuously) traded assets. The portfolio is rebalanced each month using information at *t-1*. Moreover, assets traded throughout a month, is assigned to one of the two portfolios on basis of their average daily trading volumes in NOK for the last 2 years in the market. The two-year average avoids a too frequent shift of portfolio-assets. Finally, we employ two market wide indices consisting of all the stocks in the Norwegian market with 1) equally weighted stocks and 2) market value weighted stocks. The crash in October 1987 is included. We therefore assume that a crash is normal in an equity market.

{Insert Figure 1 and 2 about here}

The Norwegian value-weighted indices are reported in Figure 1. The index shows an approximate yearly growth of 12%. The natural logarithm of total trading volume is reported in Figure 2. Note especially its strong but erratic trend in trading volume for the Norwegian thinly traded market. On average, the yearly growth in the trading volume is approximately 32,9%. We adjust for systematic location and scale effects (Gallant and Tauchen, 1992) in all time series. The log first difference of the price index is adjusted. Let ϖ denote the variable to be adjusted. Initially, the regression to the mean equation $\varpi = x \cdot \beta + u$ is fitted, where x consists of calendar variables that are most convenient for the time series and contains parameters for trends, week dummies, calendar day separation variable, month and sub-periods. To the residuals, $\hat{u}$, the variance equation model $\hat{u}^2 = x \cdot \gamma + \varepsilon$ is estimated. Next $\frac{\hat{u}^2}{\sqrt{e^{x \cdot \hat{\gamma}}}}$ is formed, leaving a series with mean zero and (approximately) unit variance given x.

Lastly, the series $\hat{\varpi} = a + b \cdot (\frac{\hat{u}}{\sqrt{e^{x\hat{\gamma}}}})$ is taken as the adjusted series, where *a* and *b* are chosen so that $\frac{1}{T} \cdot \sum_{i=1}^{T} \hat{\varpi}_i = \frac{1}{T} \cdot \sum_{i=1}^{T} \varpi_i$ and $\frac{1}{T-1} \cdot \sum_{i=1}^{T} (\hat{\varpi}_i - \overline{\varpi})^2 = \frac{1}{T-1} \cdot \sum_{i=1}^{T} (\hat{u}_i - \overline{u})^2$. The purpose of the final location and scale transformation is to aid interpretation. In particular, the unit of measurement of the adjusted series is the same as that of the original series. We do not report the result of these raw data series adjustments[xiv]. We report the raw and adjusted series for the value-weighted index in Figure 3. Table 1 reports characteristics for all the adjusted raw return series for our investigation.

{Insert Figure 3 about here}

{Insert Table 1 about here}

From Table 1 the following immediate observations can be extracted. The standard deviation of adjusted returns seems to increase as trading volume decreases. The daily maximum and minimum returns for individual assets seem to suggest that highest absolute numbers be found for the most thinly traded assets. The thinly traded portfolio does not show this characteristic due to many zero return observations. The mean returns show no clear pattern over series. The thinnest traded assets show high returns accompanied by high standard deviation.

The numbers for kurtosis and skew for the return series suggest a substantial deviation from a normal distribution. The deviation is especially strong for asset VP-6. Interestingly, the value-weighted market index also reports high absolute kurtosis and skew. Moreover, from Table 1 it seems as especially the kurtosis increases as the number of combined assets in the series increases[xv]. All series report negative skew. Hence, together the kurtosis and skews suggest too much probability mass around the mean, too little around 1-2 standard deviation from the mean and some extreme values on especially the negative side of the distribution. The kurtosis and skewness indication of non-normality is supported by the Kolmogorov-Smirnov Z-test statistic (K-S Z-test) for normality for all assets and portfolios. The ARCH (Engle, 1982), RESET (Ramsey, 1969) and BDS (Brock and Deckert, 1988 and Scheinkman, 1990) for *m = 2, 3 and 4* and $\varepsilon = 1$ test statistics report all data dependence in all series. The ARCH test suggests changing conditional volatility, the RESET test suggests non-linearity in the mean and the BDS test statistics suggest a high general nonlinear dependence in all series. For individual assets the BDS test statistic for both *m=2, 3 and 4*, increases as trading volume decreases. Moreover, note especially that where we find long non-trading periods, the BDS statistic reports highly significant values. In contrast, trading volume series report increased nonlinear dependence when trading volume increases. Overall the ARCH, RESET and BDS test statistics report a stable and strongly significant nonlinear dependence in all Norwegian return series. The RESET (Ramsey, 1969) test statistic rejects the null of linearity in only a few assets, none of the portfolios and indices. However, note that while the ARCH and BDS test statistic focus on non-linearity in both the conditional mean and variance, the RESET test statistic focus more on non-linearity in the conditional mean and parameter changes therein.

Interestingly, and in contrast to the ARCH and BDS test statistics, it seems as the RESET test reports higher values and therefore rejects linearity the more frequently an asset is traded. Together the test statistics seem to report high ARCH (non-linearity in the conditional variance) effects for thinly traded assets in contrast to low RESET (non-linearity in the conditional mean) effects. The BDS test statistic reports results in line with the ARCH test statistic, which suggests that nonlinear dependence is mostly found in the conditional variance equation. However, the results seem to suggest a need for a balancing of conditional mean and variance nonlinear dependence. Therefore, a closer look at the origin of nonlinear dependence is clearly warranted from the Norwegian raw data series. Moreover, the inspection to follow must include other characteristics of the Norwegian thinly traded equity market, which is clearly indicated from Table 1. Especially, non-normality and changing volatility are therefore all ingredients of our empirical investigation.

4 Empirical Results

Since my interest is in non-linear dependence in an ARMA-GARCH specification, the study first looks at filtering stock returns using a suitable *ARMA (p,q)* (auto-regressive and moving average) process for the conditional mean with the lag specification lengths chosen according to the Schwarz BIC Criterion (Schwarz, 1978). For the BIC choice of p and q, the seven most frequently traded assets (1-3), the three most frequently traded portfolio series and the value weighted market index BIC prefers an ARMA (0,1)[xvi] specification. The thinnest traded individual assets (asset no. 4-7), and the most thinly traded portfolio series, BIC prefer an ARMA (0,2) specification. The two equal-weighted market indices BIC prefer an ARMA (1,0) specification. Hence, the ARMA lag structure length seems to suggest dependence of thin trading.

The ARCH, RESET and BDS test statistics are all applied to the residuals from the ARMA processes with a constant conditional variance. To stay inside the boundaries for asymptotic normality of the BDS test, the statistic is computed for m in the range from 2 to 8[xvii], and $\varepsilon = 1$. The results are presented in Table 2 under the category-line υ (linear; model (1)) and ω (nonlinear; model (2)) for all series. The ARCH, RESET and the BDS test statistics all clearly suggest that the null hypothesis of i.i.d. asset returns is rejected at 1% for all series examined at all dimensions *(m)* for both linear and nonlinear specifications. Moreover, the ARCH and BDS test results for $m = 2$, 3 and 4 and $\varepsilon = 1$ is almost identical to the numbers from Table 1. Hence, the ARCH, RESET and BDS test statistics strongly reject linearity for the residuals of both linear and nonlinear ARMA conditional mean specifications, in the same manner and magnitude as it rejects linearity of the adjusted raw return series for *m* equal to 2, 3 and 4 in Table 1. These findings are consistent with the results of de Lima (1995a) from US. Moreover, thin trading and therefore non-synchronous trading and non-trading effects, seems to the increase the significance of the test statistics. Hence, the ARCH, RESET and BDS test statistics seem to report strong nonlinear dependence in series incorporating periods of zero returns. Consequently, thin trading seems to exhibit nonlinear dependence.

Our results so far seem to suggest that the nonlinear dependence in return series are strong but that nonlinear dependence in mean is small. Lee et al. (1993) raised the issue of whether the detection of nonlinear dependence in financial time series could be due to either neglected nonlinear structure in the mean or ARCH/GARCH effects (conditional heteroscedasticity). We have above found small mean nonlinear dependence. Hence, we proceed to test for nonlinear dependence from the conditional variance process; that is, model (3) and (4) from Section 2. The model we now first approach is a specification that employs a linear conditional mean equation and a nonlinear conditional variance equation.

{Insert Table 2 about here}

The GARCH (*m,n*) model for the conditional variance equation, the *m* and *n* lags are chosen based on the BIC criterion (Schwarz, 1978) of the squared residuals from an ARMA (*p,q*) process. In most cases a GARCH (1,1) is an appropriate and parsimonious representation of conditional variance equations (Bollerlev, 1986; Akigary, 1989; and Bollerslev et al., 1992). By use of the BIC criterion the highest lag representation is $m = 1$ and $n = 2$; that is, a GARCH (1,2) representation. All portfolios and indices BIC prefer a GARCH (1,1) specification. The individual assets number 2, 3, 6 and 7 BIC prefer a GARCH (1,1) specification, while the assets number 1, 4, and 5 BIC prefers a GARCH (1,2) specification for the conditional variance equation[xviii]. We investigate whether conditional heteroscedasticity is responsible for rejection of the i.i.d. hypothesis by applying the ARCH, RESET and BDS test statistics to the residuals from the BIC efficient ARMA (*p,q*) - GARCH (*m,n*) model. However, as discussed above in Section 2, the BDS test statistic is applied to the adjusted standardized residuals from the same specifications. The results are reported in Table 2 for normal residuals and in Table 3 for student-t density log-likelihood function residuals for all series under the category-lines $ln(\xi^2)$ for the linear conditional mean model (3).

In the normal case reported in Table 2, the results of applying the ARCH test statistics to the standardized residuals in line ξ, suggest that the null of constant conditional variance is rejected at 1% for all series. Therefore, filtering series through a linear ARMA-GARCH specification report non-significant ARCH effects. Moreover, for all series the RESET test fails to reject linearity against an unspecified alternative. Finally, the BDS test statistic for the adjusted standardized residuals fail to reject linearity for assets that show continuous trading (assets 1 to 4), the frequently traded portfolio and the value-weighted market index. Hence, the null of i.i.d. is rejected for (1) assets no. 5 to 7 and (2) the equal-weighted index from Oslo Stock Exchange. All the continuously traded assets no. 1 to 4, the frequently traded portfolio and the value weighted market index reject non-linearity of the residuals. This result suggests that nonlinear dependence is fully accounted for by the conditional heteroscedasticity if trading frequency show close to continuous trading (> 90%). Hence, non-synchronous trading and non-trading effects seem also for ARMA-GARCH specifications to exhibit non-linear dependence.

The student-t density case is reported in Table 3. Applying the ARCH test statistics to the estimated number of freedom standardized residuals, fail to reject the null of constant conditional variance for the residuals for all series, except the thinnest traded asset (asset no. 5 to 7). The RESET statistic fails to reject linearity for all series. Finally, the BDS statistic for dimension *m=2 to 8* reports symptoms of nonlinear dependence in the adjusted standardized residuals for asset no. 5 to 7, the thinly traded portfolio and the equal-weighted market index. However, the frequently traded asset (1 to 4), the frequently traded portfolio and the value-weighted index fail to reject linearity in the adjusted standardized residuals. Hence, applying the standardized residuals from a student-t log-likelihood function seem to produce the same nonlinear dependence in series as a normal log-likelihood function.

{Insert Table 3 about here}

Finally, we introduce nonlinear dependence in the conditional mean of an ARMA-GARCH lag specification, to see if the remaining non-linearity can be removed from the data series. Note that this model departs form the Martingale hypothesis and is described in detail in Section 2, model (4). The results are reported in the line $ln(\varpi^2)$ in Table 2 (normal) and 3 (student-t). For both Table 2 and 3 and therefore for both normal and student-t density estimations, the non-linearity in mean seems to introduce increased nonlinear dependence in the series relative to a linear ARMA-GARCH lag specification. Especially the thinnest traded assets seem to report symptoms of increased nonlinear dependence. Therefore, introduction of a non-linear mean in an ARMA-GARCH (*m,n*) estimation seems not to produce any improvements in non-linear dependence for Norwegian data series.

5 Findings from the Norwegian thinly traded equity market

The major finding of our investigation is that conditional heteroscedasticity and volatility clustering is the major cause of nonlinear dependence found in the adjusted raw return series in Table 1. For relatively frequently traded asset series nonlinear dependence seems to be filtered out in all the residuals. In fact, if we ignore the results from the thinly traded asset series none of the BDS test statistics is significant for the normal and the student-t density estimations. From these results it seems that conditional heteroscedasticity count for all non-linear dependence in the Norwegian equity market excluding thinly traded return series.

Thin trading induces non-trading and the BDS test statistic suggests that the non-synchronous trading and conditional heteroscedasticity specification does not account for all the non-linearity found in adjusted return series. The return series that rejects the null of linearity are the thinly traded individual asset series from VP-5 to VP-7, the thinly traded asset portfolio series, VP-TT and the equal-weighted index series, VP-EW. All these series where non-trading contributes strongly, fails to reject non-linearity applying the correlation integral and the BDS test statistic. However, neither ARCH nor the RESET test statistics report significant test values. Moreover, model extensions to specifications that exhibit non-linearity in mean and therefore departure form the Martingale hypothesis seems to introduce increased nonlinear dependence for especially the thinnest traded assets. Hence, introduction of a nonlinear mean seems not appropriate for any of the Norwegian data series. The nonlinear mean specification seems not to capture any of the non-trading effects. Consequently, applying the linear ARMA-GARCH filter specification we reject the independence hypothesis and fail to reject the Martingale hypothesis for all series. Moreover, our results suggest that the BDS tests statistic for i.i.d. series will always reject the i.i.d. proposition for all series exhibiting non-trading characteristics. Non-trading seems therefore to be the major contributor to the significance of the BDS test statistic and consequently ARMA-GARCH misspecification. Hence, long periods of non-trading and therefore long series of zero returns are the major source for rejection of the null of i.i.d. in stock return series applying ARMA-GARCH models. In fact, it seems as a trading proportion below 90% of total listed days implies misspecification. The BDS test statistic may suggest a need for more elaborate ARMA-GARCH models controlling for number of non-trading days applying virtual returns (Campbell et al., 1997)[xix] or stochastic volatility models, which is conditional mean independent[xx].

In summary, the main finding of this research therefore suggests that reasonable frequently traded asset series in the Norwegian equity market, seem appropriately specified by a linear conditional mean equation (ARMA(*p,q*)) and conditional heteroscedasticity applying GARCH (*m,n*) specification for the conditional volatility. All lag structure in mean and volatility should be BIC preferred. Hence, the adjusted series must be controlled for non-synchronous trading and conditional heteroscedasticity producing synchronous and conditional homoscedastic residual series. For thinly traded assets the simple and discrete time ARMA-GARCH model does not appropriately capture the non-trading effects in observed return series.

6 Summaries and Conclusions

We have found strong evidence to reject the null hypothesis of i.i.d. for Norwegian thinly traded asset returns. This finding supports the results of de Lima (1995a) that nonlinear dependence cannot be ruled out as an explanation to the dynamics of the stock returns after the 1987 crash for US data. Applying elaborate specification test statistics suggest a need to control for non-synchronous trading and conditional heteroscedasticity. Our proposed ARMA-GARCH model results (Solibakke, 2000b) suggest that the rejection of i.i.d. appear to be almost exclusively caused by non-synchronous trading and conditional heteroscedasticity. However, thinly traded assets report nonlinear dependence and consequently ARMA-GARCH model misspecification. Hence, controlling for non-synchronous trading and non-trading effects as well as conditional heteroscedasticity and volatility clustering produce well-specified ARMA-GARCH specifications for the conditional mean and volatility of relatively frequently traded asset series.

An important finding of our analysis is that modeling non-linearity in stock return series should focus on conditional heteroscedasticity and non-linearity in volatility rather than non-linear mean dependence. In fact, almost all significant non-linear dependence are ruled out using a simple BIC efficient ARMA(p,q)-GARCH (*m,n*) model for the conditional mean and volatility equations for relatively frequently traded asset series. Consequently, any non-linear mean specifications do not remedy the non-linear dependence results. Hence, we reject the independence hypothesis and fail to reject the Martingale hypothesis in the Norwegian thinly traded equity market.

Our findings suggest a strong relationship between non-trading and model misspecification. This investigation has not solved the non-trading issue, which must be left to future research. However, one way to proceed is by applying temporal aggregation (Drost & Nijman, 1993) in continuous time ARMA-GARCH specifications or applying stochastic volatility models (Gallant, Rossi and Tauchen, 1992). In the mean time, the non-trading phenomena in thinly traded series make economic implications very difficult to interpret.

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[i] For an overview of Nonlinear Dependence in Financial Data see Campbell, Lo and MacKinlay, 1997.

[ii] Conditonal dependence will through conditional mean and volatility models of raw returns, normally generate more normal residuals.

[iii] The asymmetric GARCH specification (Glosten et. al., 1993) is Lagrange Ratio preferred using both normal and student-t density maximum log- likelihood functions in the Norwegian thinly traded market.

[iv] a_0 is the estimated volatility (constant). An alternative is to specify $h_t = 1$. However, the upcoming nonlinear results in section 4 show very small changes, and do not affect the conclusions of our work.

[v] a_0 is the estimated volatility (constant). An alternative is to specify $h_t = 1$. However, the upcoming nonlinear results in section 4 show very small changes, and do not affect the conclusions of our work.

[vi] See Glosten et al. (1993).

[vii] Se Employing the nonlinear ARMA residuals don't change the BIC preferred values for *m* and *n*.

[viii] Normal log likelihood function: $-0.5 \cdot \ln(2 \cdot \pi) + \varepsilon / h)^2 + 2 \cdot \ln(h)$; where ε is the residuals and *h* is the conditional variance.

[ix] Student-t log likelihood function: $C - 0.5 \cdot \ln(h) - ((\eta + 1) / 2 \cdot \ln(1 + \varepsilon^2 / (\eta - 2) \cdot h))$;
where *C* is a constant, ε is the residuals, *h* is the conditional variance and η is the degree of

freedom parameter.

[x] See also Tsay (1986), Spanos (1986) and Lee et al. (1993).

[xi] If $\{\mu_t\}$ is a strictly stationary, absolutely stochastic process, the integral defined below exists.

[xii] The simulation is based on 2000 replications, each with 1000 observations

[xiii] We have chosen a Student-t distribution as it has been found to suit Norwegian equity data well (Solibakke, 1999).

[xiv] The results are readily available from the author upon request.

[xv] Often named the mixture of distributions hypothesis, which maintains that asset returns exhibit leptokurtosis because they are really a combination of returns distributions.

[xvi] ARMA(0,1) is found to model non-synchronous trading (Lo and MacKinlay, 1990).

[xvii] The maximum choice of m, 8, is chosen so that T/m is higher than 200 (Brock et al., 1991)

[xviii] The result implies 4 different ARMA-GARCH models to estimate for assets and portfolios. The BHHH algorithm (Berndt et al., 1974) is employed for estimation.

[xix] At Molde College we apply virtual returns (Campbell, 1997) and the number of non-trading days in a continuous time ARMA-GARCH lag specification to model non-trading effects. The premature results seem encouraging.

[xx] See for example the SNP methodology of Gallant, Rossi and Tauchen, 1992.