Chapter III

Non-synchronous trading and Volatility Clustering in Thinly Traded Markets

Abstract

ARMA-GARCH lag specification is employed to fit a model exhibiting non-synchronous trading and volatility clustering for the Norwegian thinly traded equity market. In particular, we investigate characteristics of the conditional mean and conditional volatility inhibited in thinly traded equity markets. We employ trading volume as a proxy measure for trading frequency. Low to no trading volume induces thin trading and non-trading effects while a relative high trading frequency induces continuous trading. Our main objective is to investigate trading frequency differences in autocorrelation and cross-autocorrelation in the mean and volatility clustering as well as any symptoms of data dependencies in the model residuals, which imply ARMA-GARCH model misspecification. We employ BIC efficient ARMA-GARCH lag specifications for the conditional mean and volatility and introduce relevant mean and volatility parameter measures that are well known from the changing volatility literature. Our results report consistent mean and volatility patterns over increasing trading frequency series. Non-synchronous trading and non-trading effects show a consistent pattern in autocorrelation and cross-autocorrelation for the conditional mean and volatility clustering exhibits a consistent pattern in past shocks, past conditional volatility, persistence and weight to long-run average volatility. In contrast to the more relatively frequently traded asset series the most thinly traded series report low and insignificant asymmetric volatility. However, specification tests suggest data dependence for the most thinly traded series, which seems to be prolonged into the equal-weighted index. Hence, due to serial correlation and data dependence in model residuals the ARMA-GARCH lag specifications are only appropriate for relatively frequently traded series.

Classification:

Keywords: Non-synchronous trading and non-trading effects, volatility clustering, trading frequency, ARMA-GARCH lag specifications

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1 Introduction

ARMA-GARCH lag specification for the conditional mean and volatility is employed for a non-synchronous trading and changing volatility model characterising the thinly traded Norwegian equity market. Solibakke (2000a) applies autocorrelation results from Campbell et al. (1997) and show that the Norwegian equity market is a relative thin market compared to more elaborate markets. Solibakke (2000a) also show that the Norwegian market exhibit thinly traded series relative to continuously traded series[1]. Hence, the main motivation is to investigate any differences in the conditional mean and volatility from thinly to continuously traded series. We apply trading volume as a proxy measure for trading frequency. We employ observed return series and non-trading and therefore zero returns, is characterized by zero trading volume in Norwegian Kroner (NOK). As the Norwegian market exhibits long return series with zero trading volume, trading volume proxies well for non-synchronous trading and non-trading effects induced by the zero return series. To establish trading frequency series we form four portfolios based on trading volume in NOK. These four series and an equal- and a value-weighted market index series become the main empirical investigation focus. We investigate the dynamics in return series that exhibit an increasing trading frequency, employing ARMA-GARCH methodology to model the conditional mean and volatility processes. ARMA-GARCH estimations from thinly to continuously traded series may give new and interesting information of non-synchronous trading and non-trading effects as well as volatility clustering. The investigation are especially interested in effects from autocorrelation and cross-autocorrelation in the conditional mean and shocks, autocorrelation, persistence and asymmetry in the conditional volatility. Hence, this investigation studies the relationships between trading frequency and conditional mean and volatility dynamics in an estimation context that control for non-synchronous trading and conditional heteroscedasticity. Finally, to complete the model features, we incorporate asymmetric volatility as well as a measure of residual risk from the conditional volatility to the mean (in-Mean). To my knowledge the focus of trading frequency and BIC preferred lead and lag structures for the conditional mean and volatility specifications are new and are not previously been carried out in international studies.

The portfolio series are organized based on historic trading volume and are rebalanced monthly, where the thinnest traded portfolio captures very thin trading, the intermediate thinly traded portfolio captures dynamics for thinly traded series and the two frequently traded portfolios capture medium to frequent (continuous) trading. All time series are adjusted for systematic scale and location effects and a correct lag structure for the conditional mean and volatility are achieved by a BIC (Schwarz, 1978) preferred ARMA-GARCH-in-Mean lag specification. Note that the univariate ARMA-GARCH-in-Mean specification represents a

[1] See chapter 1.1 of my dissertation (Solibakke, 2000a) for a definition and classification of thin trading in the Norwegian equity market.

departure from Brownian Motions (Bachelier, 1964) and random walk. The specification explicitly allows for predictability measures in both mean and volatility processes.

We believe that the contribution of this paper is a higher understanding of the workings of mean and volatility processes in thinly traded markets, where non-synchronous trading and non-trading effects as well as volatility clustering may contribute significantly to the dynamics of asset pricing. The specification contributes by the following model features across varying trading frequency. Firstly, the specification seeks consistent coefficient differences in the conditional mean equation; that is, autocorrelation and cross-autocorrelation. Secondly, consistent and significant coefficient differences in the volatility equations may contribute to a higher understanding of lagged shocks effects, auto-correlated and asymmetric volatility and the weight to long-run average volatility. Thirdly, as the degree of leptokurtosis in residuals measures the departure from the normal distribution, any systematic and significant coefficient differences may contribute to a higher understanding of non-normal returns. Fourthly, as we employ the Bayes information criterion (Schwarz, 1978) (BIC) for lag specification in both the mean and the volatility equations, efficient ARMA-GARCH specification is obtained in both mean and volatility. Any change in lag structures may offer new and higher understanding of mean and volatility dynamics for thinly traded markets. Fifthly, as we perform elaborate specification test statistics and single and joint tests for volatility prediction biases, any misspecifications will be reported.

We follow an expansion path starting from adjusted raw returns and eventually specify both ARMA (mean) and GARCH (volatility) specifications for all the employed data series. As these Norwegian equity series contain assets that show thin trading relative to continuous trading, this investigation may contribute substantially to the international non-synchronous trading[2] and changing volatility literature. Consequently, the ARMA-GARCH specifications for the Norwegian equity market may characterise non-synchronous trading and non-trading effects as well as volatility clustering across varying trading frequency series not earlier shown in international finance.

The remainder for this paper is therefore organised as follows. Section 2 gives a literature overview of changing volatility, non-synchronous trading and volatility clustering. Section 3 defines the data and describes a general adjustment procedure for systematic location and scale effects in time series. Section 4 specifies the ARMA lag specification for the conditional mean and the GARCH lag specification for the conditional volatility, employing the BIC (Schwarz, 1978) methodology to ensure efficiency. Section 5 reports the empirical results and Section 6 reports the findings from the analysis. Finally, section 7 summarises and concludes our findings.

[2] See Solibakke (2000a) for non-trading characteristics for individual assets in the Norwegian equity market.

2 Literature overview

If a subordinated stochastic volatility model determines asset returns, then returns during periods of non-synchronous trading and non-trading would differ from returns during periods of synchronous and continuous trading. Assuming trading frequency is a proxy for (non)-synchronous trading effects, we may hypothesise that trading periods containing low or no trading volume is characterised by mean and volatility processes different from processes in trading periods containing high trading frequency. Clark (1973) develops a subordinated stochastic process model for speculative price series. He argues that observed daily price changes are driven by two components; (1) a subordinated (or conditional) price change process and (2) a driving (or operational time) process. Clark found that the variability of the observed price process differs from one chronological time period to another, depending on the volume of transactions. Hence, a mix of finite volatility processes may describe price change series. Tauchen and Pitts (1983) have later refined this research assuming a stochastic volatility process. Moreover, Gallant and Tauchen (1996) employ efficient method of moments to estimate stochastic volatility models with diagnostics. They find that stochastic volatility models describe market characteristics well allowing for autocorrelation in both the mean and volatility processes.

Other research supports the mixture of distribution's hypothesis by testing subordinated stochastic process models of price change series and trading volume series.[3] [4] Harris (1989) argues that observed properties of daily data are a consequence of similar properties of transaction data. Because each transaction price change is leptokurtic, leptokurtosis is a result of daily price changes when transaction data are aggregated to obtain daily data. Using a mixture of distribution model for daily data that is conditioned on the arrival of information in a given day, Harris finds kurtosis, skews and heteroscedasticity in daily price changes. His results also suggest that the daily transactions count may be a useful instrumental variable of estimating unobserved realisations of stochastic price variances. However, the system is still incomplete, as the dynamic properties of the information arrival process, which is assumed to drive return, volatility and volume, remain unspecified. Hence, in recent years we find that many analytical models of information arrival find that returns and trading-volume are co-determined. For example, Admati and Pfleiderer (1988) model the effects of private information on order flow and that the trades of several classes of investors (informed traders and discretionary liquidity traders) will tend to cluster. This clustering of trades causes return variance to be highest during periods of active trading. In an alternative approach to the relation of information arrival, volume, return and variances, Ross (1989) assumes that

[3] Epps and Epps (1976), Morgan (1976), Westerfield (1976), and Tauchen and Pitts (1983).
[4] Theoretically, these processes can be derived as discrete time approximations to the solution of the option valuation problem when the volatility of the underlying asset price is stochastic. Research in this vein has been carried out by, for example, Scott (1987), Wiggins (1987), Chesney and Scott (1989) and Melino and Turnbull (1990).

information arrives according to a martingale process and, though no arbitrage conditions, demonstrates that return variances are proportional to the rate of information flow. In this case, price change when there is new information is coincident with trades.

Return volatility and trading volume will be related if transaction arrivals are related to the flow of information in the model. Hence, a growing body of empirical evidence supports the joint determination of return variance and trading volume.[5] However, while international research focuses on short-term conditional heteroscedasticity in bivariate asset trading frequency and return estimation (see for example SNP[6] estimation in Gallant, Rossi and Tauchen, 1992), we focus here on systematic and consistent differences in lag structures and coefficient changes in conditional mean and volatility equations for thinly and frequently traded assets employing univariate ARMA-GARCH-in-Mean lag specifications. Hence, in contrast to Lamoureux and Lastrapes (1990), trading volume series are not directly modelled in the conditional volatility process. In our univariate investigation we aim to find consistent lag and coefficient differences in the conditional mean and volatility processes for series showing an increasing trading frequency.

The model specifications focus on changes in lag structures as well as changes in coefficients in the conditional mean and volatility across varying trading frequency series for the Norwegian market. Our focus will be on differences in non-synchronous trading and non-trading effects as well as conditional heteroscedasticity and volatility clustering. Intuitive thinking suggest that an asset that reports non-trading responds to new information with a time lag. These lagged responses may induce biases in the moments and co-moments of daily return series. The serial correlation may influence tests of predictability and non-linearity as well as volatility risk and expected returns. The first to recognize the importance of non-synchronous trading was Fisher (1966). Campbell et al. (1997) reviews and extends existing theory. They show that large stocks tend to lead those of smaller stocks, which suggest that non-synchronous trading may be a source of correlation. However, they also find that the magnitudes for the autocorrelations imply an implausible level of non-trading and therefore leads them to the conclusion that non-trading is only responsible for some of the autocorrelation. Moreover, applying estimated non-trading probabilities from daily autocorrelations (Campbell et al. 1997) they find little support for non-synchronous trading and non-trading effects as an important source of serial correlation in the returns for common stock over daily and longer frequencies[7].

[5] See Barclay, Litzenberger and Warner (1990), Gallant, Rossi and Tauchen (1992) and Andersen, 1994.
[6] A Semi-Non-Parametric Score Generator (Gallant and Tauchen, 1989)
[7] See also Boudoukh, Richardson and Whitelaw (1995), Mech (1993) and Sias and Starks (1994). All three papers conclude that non-trading cannot completely account for the observed autocorrelations.

3 Empirical data and Methodologies

3. 1 Empirical Data

The study employ daily returns and trading volume for individual Norwegian stocks spanning the period from October 1983 to February 1994. Daily return series are defined as $ln(p_{i,t}/p_{i,t-1})$, where $p_{i,t}$ is the daily closing price for asset i at time t. Trading volume is defined as the total transaction volume in NOK at day t for asset i including external trading (trading outside the organised market). The individual shares are grouped into portfolios at period t based on trading volume series in the information set at $t-1$, Ω_{t-1}. We rebalance the portfolios each month and to avoid a too frequent shift of component stocks among the asset portfolios we employ the average daily trading volume for the last two years. Two years of daily volume is chosen to obtain a time overlap of 95% for each portfolio restructuring[8]. Hence, assets are arranged into portfolios based on changes in trading volume over a considerable time period. We emerge from this exercise with four series; a thinly traded series that contains the most thinly traded assets (Portfolio 1), an intermediate thinly traded series (Portfolio 2), an intermediate frequently traded series (Portfolio 3), and finally a frequently traded series that contains the most frequently traded assets (Portfolio 4). In this exercise we have employed all assets in the Norwegian thinly traded market and on average all series therefore contain at least 25 assets. Moreover, we divide the time periods into two sub-samples; (1) a time period before the crash in October 1987 (1019 daily observations), (2) a time period after the crash (1546 daily observations), and (3) a time period for the entire 10 years time period 1983-1994 (2611 daily observations)[9]. Note that to keep the paper within reasonable limits we only report results for the entire period. Relevant sub-period results are described in footnotes. Note also that the crash is included for the entire period 1983-94, which induce that market dumps are considered normal in equity market. Moreover, we include two market wide indices consisting of all the stocks in the Norwegian market with (1) equally weighted stocks and (2) market value weighted stocks. We include these indices to aim to recognise patterns from trading volume portfolios on the index level. Therefore, this high frequency time series database gives us potentially 2611 observations for each portfolio and index and is our main vehicle to achieve our objectives for the investigation of thinly traded markets.

For all time series we employ the procedures described by Gallant, Rossi and Tauchen (1992) to adjust for systematic location and scale effects in all six return series[10]. The procedure gives us series that become more homogenous allowing us to focus on the day-to-day

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[8] Moreover, the first trading volume observation we have been able to collect for all assets in the Norwegian equity market is registered 01.09.81.

[9] The crash period in this study is defined as the two months October and November in 1987. The estimations and specifications results for the two sub-periods are not reported based on space considerations. However, all results are available from the author upon request.

[10] The scale and location results for all six series are not reported but are available from the author upon request.

dynamic structure under an assumption of stationary series without any disturbance to mean and volatility characteristics. To show these properties from the procedures we report the value-weighted index and the natural logarithm of the total trading volume in Figure 1 and 2, respectively. The index shows an approximate yearly growth of 12%. In particular, note the strong and erratic trend in trading volume for the Norwegian Market. On average, the growth in the trading volume in NOK is approximately 32,9% per year.

{INSERT FIGURE 1 OG 2 ABOUT HERE}

{INSERT TABLE 1 ABOUT HERE}

The characteristics of the adjusted portfolio and market index series are reported in Table 1. Table 1 shows that the mean returns are highest for the thinnest traded series and are accompanied by the highest standard deviation. Hence, both expected return and total risk are at its peak in these series. The most frequently traded series show lower return and standard deviation relative to the most thinly traded series. The two intermediately traded series show characteristics between the thinnest and the continuously traded series. The most frequently traded series show lowest minimum and highest maximum daily return, which also induce high absolute skew[11] (negative tails) and high kurtosis[12]. The ARCH test statistic is a test of changing volatility in the return series. All series report highly significant changing volatility and suggests a need for ARCH/GARCH specification of the second moments. The RESET (Ramsey,1969) test statistic suggests non-linearity in the mean for all series. The BDS (Brock et al. (1988, 1991), Deckert (1991) and Scheinkman (1990)) test statistic suggests general non-linearity for all series at all dimensions (*m*) and for ε equal to one[13]. Finally, the K-S Z-test suggests deviation form normally distributed adjusted returns for all series. The thinnest traded series show lowest deviation while the most frequently traded series show highest non-normal returns. The skew and kurtosis numbers confirms all these K-S Z-test results.

The market index series show as expected from portfolio literature, lower standard deviation. The numbers for the mean, maximum and minimum returns show no extraordinary pattern relative to the other portfolio series. As for the other series, the indices report changing volatility (ARCH), non-linearity in the mean (RESET) and general non-linear dependence (BDS)[14]. The numbers for skew and kurtosis suggest considerable deviation from normality

[11] Skew: A measure of the thickness of the tails of a distribution
[12] Kurtosis: A measure of the asymmetry of a distribution.
[13] Calculated as ($\varepsilon \cdot$ standard deviation). ε equal to 0.5, 1.5 or 2 does not materially change our conclusions.
[14] Sub-periods report (1) a reduced mean return and (2) an increased volatility story after the crash in October 1987. The average reduction in mean return for the four trading volume portfolios and the two market indices is approximately 107% and 82.8%, respectively, and the average increases in volatility is 54.8% and 38.6%, respectively. The thinnest traded assets have the highest increase in volatility. Moreover, the last sub-period (1987-94) produces

and are higher in absolute values than for the other trading frequency series. In fact, from the kurtosis and skew numbers the strongest deviation from normally distributed returns is found for the two index series, which is confirmed by the K-S Z-test statistic.

3.2 The ARMA-GARCH-in-Mean methodology

Table 1 reports autocorrelation, changing volatility, non-linearity and systematic leptokurtosis in the return distributions for all series. Hence, Table 1 suggests non-normal returns, ARMA effects in mean, ARCH/GARCH effects in volatility and a need to control for serial correlation and data dependence in model residuals (misspecification). Table 2 reports mean, standard deviation and autocorrelation for daily returns and squared returns in order to accomplish model specification. The ARMA-GARCH lag specification is strongly enhanced as the autocorrelation for returns and squared returns show clear patterns. In fact, the autocorrelation structure report the necessary underlying material for the conditional mean and volatility specification in the ARMA-GARCH methodology found in ENGLE (1982), Bollerslev (1986, 1987) and Engle and Bollerslev (1986). Moreover, Table 2 reports the mean and standard deviation for all series, which enhance the setting of starting values for serial correlation estimation in both mean and volatility. The ARMA lag specification models non-synchronous trading and non-trading effects while the GARCH lag specification models conditional heteroscedasticity and volatility clustering. International literature applying ARMA-GARCH specifications have shown that these models are able to account for many of the lag structures found in observed mean and volatility processes.

To obtain the most efficient ARMA-GARCH lag specification for the conditional mean and volatility, we perform a specification procedure that accommodates the characteristics of the return series. We approach the model’s lag specifications for the conditional mean and volatility for our return series below. Applying elaborate specification test statistics to the resulting lag structure residuals will determine whether the ARMA-GARCH model is able to accommodate the observed market characteristics.

3.2.1 The conditional mean specification

For the conditional mean specification, Table 2 reports the autocorrelation structure up to lag 6 for the adjusted daily return series. We find negative serial correlation at lag one for the thinly traded series. In contrast, the frequently traded series report significant positive serial correlation. The general picture is therefore negative serial correlation for thinly traded series and positive serial correlation for frequently traded series. The correlation structure suggests that thinly traded series show mean reversion and therefore negative time dependence

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higher positive kurtosis, higher negative skew and the non-linear dependence seem to increase.

(Poterba and Summers, 1988 and Fama & French, 1988). However, thin trading imply series of zero returns and may therefore induce biases to the moments of the return series, which may produce spurious autocorrelation. Table 2 reports that the magnitudes of the serial correlation coefficients decay very fast at higher orders. By applying the above reported correlation structure and applying a procedure described by Box and Jenkins (1976) we may establish a parsimonious representation of the conditional mean structure. As we want to establish the model specification of an *ARMA (p,q)* process we employ the Schwarz criterion (BIC) to determine *p* and *q*. The BIC criterion (Schwarz, 1978) is computed as $BIC(p,q) = \ln\sigma^2 + (p+q)T^{-1}\cdot\ln T$, where σ^2 is the estimated error variance and *T* is the number of time periods employed. We prefer small values of the criterion. The criterion reward good fits as represented by small $\ln\sigma^2$ and uses the term $(p+q)T^{-1}\cdot\ln T$ to penalise good fits that is got by means of excessively rich parameterisations. The criterion is conservative in that it selects sparser parameterisations than the Akaike information criterion (Akaike, 1969) (AIC), which uses the penalty term $2\cdot(p+q)T^{-1}$ instead of $(p+q)T^{-1}\cdot\ln T$. Schwarz is also conservative in the sense that it is at the high end of the permissible range of penalty terms in certain model selection settings (Potscher, 1989). Between these two extremes is the Hannan and Quinn (Hannan, 1987) criterion. The usual suggestion is to use the Schwarz BIC criterion to move along an upward expansion path until an adequate model is determined. Hence, we employ the procedure $BIC(p_1,q_1) = \min BIC(p,q), p\in P, q\in Q.$

{INSERT TABLE 2 ABOUT HERE}

Table 3 reports the computed BIC, AIC and HQ criteria for three ARMA model specifications for all series. Table 3 shows that the three most frequently traded series and the value-weighted market index all prefer an *ARMA (0,1)* lag specification for the conditional mean. Hence, these four series prefer an autocorrelation specification that employs a one period lagged moving average specification (*MA (1)*). The thinnest traded portfolio BIC prefers an *ARMA (0,2)* lag specification for the conditional mean. This lag specification suggests severe non-trading effects modelled by a two periods lagged moving average specification (MA(2)). Finally, the equal-weighted index prefers an *ARMA (1,0)* lag specification for the conditional mean. This specification suggests that combined series from thinly and continuously traded series seem to prefer an autoregressive specification (AR(1)). Moreover, interestingly, thin trading and non-trading effects seem to affect the mean specification differently in combined series containing similar assets versus all equity equal-weighted index series. Our preferred conditional mean specification for all series therefore becomes

$$R_{i,t} = \mu_i + \phi_{i,1}\cdot R_{i,t-1} + \varepsilon_{i,t} - \theta_{i,1}\cdot\varepsilon_{i,t-1} - \theta_{i,2}\cdot\varepsilon_{i,t-2} \text{ for } i = 1, 2, 3, 4, EM, VM. \qquad (1)$$

where R_i *is* return for portfolio series *i*, the ϕ_i coefficient is equal to zero for all *i* except *i = EM*, the $\theta_{i,1}$ is equal to zero for *i=EM* and the $\theta_{i,2}$ is equal to zero for *i=2,3,4,EM and VM.* We are now able to estimate the mean structure employing standard *ARMA (p,q)* methodology, where *p* and *q* are BIC preferred.

{INSERT TABLE 3 ABOUT HERE}

The estimated coefficients of the *ARMA (p,q)* models of the conditional mean are reported in Table 4. The auto-regressive coefficient, $\phi_{i,1}$ and the moving average coefficients $\theta_{i,1}$ and $\theta_{i,2}$ captures the first and second order serial correlation for all the return series. Table 4 reports strong autocorrelation for the conditional mean in the Norwegian equity market and suggest considerable predictability in return series.

{INSERT TABLE 4 ABOUT HERE}

The reported autocorrelation and distribution characteristics for the residuals (ε) reported in Table 5, suggest that the *ARMA (p,q)* specification appropriately specify the conditional mean. Only the most frequently traded series may suggest model misspecification by the $Q_i(6)$ statistic (Box and Jenkins, 1976). None of the other series shows significant autocorrelation for the residuals up to lag 6. Hence, the BIC preferred *ARMA (p,q)* models seem to provide a well-specified form for the conditional mean process in the Norwegian market. Moreover, the numbers for skew and kurtosis are reduced relative to the same numbers for adjusted raw data series. However, the ARCH (6) test statistics rejects strongly conditional homoscedasticity, the RESET test statistic rejects linearity in the mean and the BDS test statistic rejects identically and independently (iid) distributed residuals. In fact, the ARCH, the RESET and the BDS test statistics are mostly maintained from the adjusted raw series at all dimensions. Hence, the data dependence reported in Table 1 and 5, may originate from the conditional volatility process. We model the conditional volatility lag structure below.

{INSERT TABLE 5 ABOUT HERE}

3.2.2 The conditional volatility specification

Table 5 above reports the serial-correlation structure in the residuals and squared residuals from the *ARMA (p,q) lag* specification of return series, where *p* and *q* are BIC (Schwarz, 1978) preferred. The standardised residuals in Table 5 show close to zero autocorrelation and suggest an appropriate conditional mean specification. However, Table 5 finds strong evidence of autocorrelation among the squared residuals (ε^2). This empirical finding lends strong support to an ARCH/GARCH specification for the conditional volatility process. To achieve a lag specification for the conditional volatility process we employ the applied test for *ARCH* effects described in Table 1. Engle (1982) shows that a test of the null hypothesis that $\varepsilon_{i,t}$ has a constant conditional variance against the alternative that the *ARMA* theory follows through. This implies that by employing the squared residual $\varepsilon_{i,t}^{2}$ we can identify *u* and *n* in an ARMA (*u,n*) specifcation for the conditional variance by applying the same methodology as conditional mean ARMA (p,q) modelling in the previous section. Hence, Table 6 reports the BIC, HQ and AIC for ARMA (u,n) models of the squared residuals from BIC preferred ARMA (*u,n*) models of the conditional mean process for all series. For all series the autocorrelation lag structure in the squared ARMA (*u,n*) residuals is consistent with an ARMA (*1,1*) model. Hence, we will use the conditional variance equation

$$h_{i,t} = m_{i,0} + a_{i,1} \cdot \varepsilon_{i,t-1}^{2} + b_{i,1} \cdot h_{i,t-1}, \qquad \text{for } i = 1, 2, 3, 4, EM, VM \qquad (2)$$

which is known as the GARCH (*1,1*) specification[15] for the conditional volatility[16]. In this model, the coefficient $a_{i,1}$ measures the tendency of the conditional variance to cluster, the $b_{i,1}$ measures the autocorrelation in conditional volatility, while the coefficients $a_{i,1}$ and $b_{i,1}$ together measures the degree of persistence in the conditional variance process. For a stable GARCH (1,1) process we require that $a_{i,1} + b_{i,1} < 1$. Otherwise, the wight applied to the long-term variance is negative. The weight is $\delta_{i,0} = 1 - (a_{i,1} + b_{i,1})$ and the long-term variance is $V_i = m_{i,0} / \delta_{i,0}$.

{INSERT TABLE 6 ABOUT HERE}

3.2.3 In-Mean, Cross Portfolio Effects and Asymmetric Volatility

We include $\sqrt{h_{i,t}}$ in the mean equation in an attempt to incorporate a measure of risk into the return generating process. We therefore induce a measure of residual risk (Lehmann, 1990) into the model. We also include $\gamma_{i,j}$ ($i \neq j$) in the conditional mean to control for any cross series effects of the type identified by Lo and MacKinlay (1990). Finally, we include a coefficient for asymmetric volatility in the conditional variance equation (Nelson, 1991, Glosten et al., 1993). We apply the methodology of Glosten et al. (1993) to model asymmetric volatility[17] in the conditional variance equation (λ_i). Finally, we assume an innovation ε_t that follows a conditional student-t distribution[18] to accommodate leptokurtosis, which we observe in Table 1 and 4 for all the series' return distributions.

[15] For applications see Bollerslev, Chou and Kroner (1992).
[16] The GARCH(1,1) specification was introduced by Bollerslev (1986, 1987) and seems to be the major specification for GARCH(m,n) models in international finance.
[17] For reference purposes we will denote this asymmetric model for GARCH-GJR. Note also that the GJR specification is Lagrange Ratio Test preferred for all series relative to an exponential GARCH lag specification (Nelson, 1991).
[18] The number of freedoms is estimated.

We employ the BHHH[19] algorithm for estimation in GAUSS[20] for all series. The final iteration employs the Newton-Raphson[21] algorithm to extract all information from the Hessian matrix. Hence, the t-ratios are based on the sum of the *kxk* matrix of second differentials over *n* observations.

4 Empirical results

Maximum likelihood estimates of the parameters for the BIC preferred ARMA-GARCH lag specifications for the conditional mean and volatility are reported in Table 6 for all series. Among the four trading frequency series, only the most frequently traded series report a significant and positive α_i coefficient in the mean equation. The index series report significant and positive α_i coefficients.

{INSERT TABLE 7 ABOUT HERE}

Autocorrelation is statistical significant in all series. The autocorrelation moves from significant positive coefficients and therefore negative serial correlation for the thinly traded series to significant negative coefficients and therefore positive serial correlation for the frequently traded series. The result suggests that assets in thinly and frequently traded portfolios exert different price adjustment mechanisms. The thinly traded assets seem to overreact from shocks at *t-1* and therefore next period at *t* reverse this overreaction and move prices back to a new and now correct price (mean reversion). The positive autocorrelation (θ_1) coefficients for the frequently traded assets suggest an adjustment to new information at both *t* and *t+1*. In both cases prices do not adjust immediately to new information. However, depending on trading frequency, asset prices either overreact and reverse or adjust slowly over several days. However, be aware of the series of zero returns in thinly traded assets, which may induce spurious autocorrelation in theses series.

We find cross-autocorrelation ($\gamma_{i,j}$) (Lo and MacKinlay, 1990) for all series except for the most frequently traded series. The pattern in the cross-autocorrelation implies mainly influence from series that show more frequent trading. However, also here thinly traded series induce spurious cross-autocorrelation. The estimated β_i coefficients on the GARCH-in-Mean terms show no significant “mean” effects. This is also true for the market indices. The degree of freedom coefficients (υ_i) suggest thick distribution tails. For all series the coefficients are strongly significant and show values ranging between 5 and 6.5.

Among the estimated conditional variance coefficients, which are all strongly significant we find a clear pattern. The past squared errors (a_1) have more influence over the conditional

[19] The BHHH algorithm is described in Berndt, Hall, Hall and Hausmann (1974).
[20] Gauss is a programming and estimation tool from Aptech Systems
[21] The Newton-Raphson algorithm estimates the Hessian matrix directly.

variance of the frequently traded portfolios than they do over the conditional variance of the thinly traded portfolios. The two market indices seem to report shock effects in line with the effects of the two frequently traded portfolios. In contrast, the past conditional variance (b_1) exerts a greater influence over the current conditional variance in the case of the most thinly traded series. Also for the conditional variance the two market indices seem to follow the results from the two frequently traded portfolios for past conditional variance. Hence, the combination of these two features of the conditional variance suggest that although shocks to the volatility of thinly traded portfolios have less impact than shocks to the volatility of frequently traded portfolios, they are much more persistent. Finally, for all portfolios and indices we find a negative asymmetric volatility coefficient (λ_i), which imply higher volatility from negative shocks in all series. The negative asymmetric volatility is insignificant for the most thinly traded series but increases in size and significance as trading frequency increases.

To test the validity of our results, we perform several model specification tests for all six return series. As a first specification test, we calculate the sixth order Ljung and Box (1978) statistic for the standardised residuals[22] (Q) and squared standardised residuals (Q^2) for all six series. For all series Table 7 shows no significant evidence of serial correlation in the standardised residuals ($Q(6)$) at 1%. However, for the squared residuals up to lag 6 ($Q^2(6)$) the thinly traded series report significant autocorrelation, while all other series report insignificant autocorrelation. The numbers for kurtosis and skews for the standardised residuals are lower in absolute values for all series. Hence, the ARMA-GARCH filter suggests clearly more normal residuals. This result is confirmed by the K-S Z-test statistic (not reported).

Table 8 report extended model specification tests. Table 8 reports the ARCH, RESET and BDS test statistic for the BIC preferred ARMA-GARCH standardised residuals (ε) and adjusted standardised residuals[23] ($ln(\varepsilon^2)$). The ARCH test statistic reports conditional homoscedasticity for all series except the most thinly traded series. The RESET test statistic cannot reject linearity in the mean for any series. The BDS test statistic rejects identically and independently distributed residuals (iid) for the most thinly traded series and the equal-weighted market index. Hence, the model specification test statistics suggest that the ARMA-GARCH model seems to capture most of the market dynamics appropriately. However, for thinly traded series the ARCH and BDS test statistics suggest a wrongly specified model. The result induce that the ARMA-GARCH model does not appropriately describe thinly traded series and long series of zero returns. Furthermore, the inclusion of the non-linear dependent ARMA-GARCH residual and thinly traded series into the index series seems to induce a wrongly specified model also for the equal-weighted market index.

{INSERT TABLE 8 ABOUT HERE}

[22] Standardised residuals are calculated as $\varepsilon_t/\sqrt{(h_t * \upsilon_i/(\upsilon_i-2))}$ where υ_i is the degree of freedom in the student-t distribution.

Finally, Table 9 reports three simple bias tests and one joint test (Engle and Ng, 1993). Table 9 from column 2 to 6, report significant ($\varepsilon_{t-1} \cdot S_{t-1}$) biases. Hence, bad news is not very well predicted by our model. Especially, the thinnest traded portfolio suggests that bad news is badly predicted. Moreover, the joint bias test statistic in column 7 and 8, reports significant prediction biases for the most thinly traded series.

{INSERT TABLE 9 ABOUT HERE}

5 Findings from the Norwegian thinly traded market

The main focus of this investigation is characteristics in thinly traded markets, especially non-synchronous trading and non-trading effects as well as conditional heteroscedasticity and volatility clustering. The Norwegian market exhibit characteristics in trading volume (NOK) that make it possible to establish asset portfolio series that contains the desired trading frequency characteristics. Hence, the investigation looks for trading volume characteristics in the mean and volatility equations as well as in the overall model specification and discuss implications for market dynamics in thinly traded markets.

Only the most frequently traded series and the two market indices report significant positive drift, while the three more thinly traded portfolios report non-significant drift. These results together with the significant and positive α_i coefficients for the two market indices, suggest that a significant and positive drift may solely originate from continuously traded series. Moreover, it seems to be the case that series exhibiting non-synchronous trading and non-trading characteristics reject positive drift. For an investor in the Norwegian market our results imply that in a long hold strategy, thinly traded series should be avoided and should only involve continuously traded assets. Furthermore, as the most thinly traded assets also imply the high non-synchronous trading and non-trading effects, the assets may induce high spurious autocorrelation and cross-autocorrelation. These non-synchronous trading and non-trading effects may also influence the drift coefficients for these assets. The direction of the influence may be difficult to classify, but as zero return will be registered in a non-trading period, the drift will probably be influenced towards a zero drift coefficient[24].

Autocorrelation is found in all series, which imply substantial predictability in asset returns for the thinly traded Norwegian market. The thinly traded series report strong negative autocorrelation while the frequently traded series and the indices exhibit strong positive

[23] See deLima, Pedro (1995a,b)
[24] Solibakke (2000) shows that the drift becomes more positive by applying virtual returns and a continuous time GARCH model.

autocorrelation[25]. However, returns for thinly traded series may contain characteristics of non-synchronous trading and non-trading effects, which may cause spurious autocorrelation as discussed in Campbell et al. (1997). Hence, our results for thinly traded series implying overreaction and reversion, may originate from spurious autocorrelation, which stem from many zero return observation. The validity of autocorrelation results for thinly traded series may therefore be disputed. For the frequently traded series the positive serial-correlation coefficient suggests adjustment to new information that may take several days. It is well known from international literature that positive dependence (Taylor, 1986/2000) in assets returns is more often found than negative dependence. Hence, collectively, the serial-correlation coefficients imply substantial predictability among all the return series. This predictability results for frequently traded series are not disputed while thinly traded series may show spurious predictability[26]. Moreover and probably very important for investors in the Norwegian market, by employing frequently traded assets in portfolio construction and prediction, our results seem to suggest a consistent short run predictability of asset returns. Note that the reported negative autocorrelation for thinly traded series may be spurious and may distort the predictability in these series. Hence, also applying estimation results form 1987 to 1994, the overall predictability may therefore be illusionary for the Norwegian market.

We find significant cross-autocorrelation among trading volume series. The cross portfolio results therefore strongly indicate that the thinly traded Norwegian market show return effects from more frequently traded series into more thinly traded series. That is, the result suggests that thinly traded series adjust to new information with a lag to more frequently traded series (γ_j). Hence, new information is incorporated into assets starting with the most frequently traded assets and then with a lag, moved into more thinly traded assets. Hence, investors may therefore follow the following procedure to obtain a long run profit. Study carefully the most frequently traded asset within an industry. When these assets move up or down take appropriate positions (long or short) in more thinly traded assets. The asset position must be constantly monitored and may be expensive owing to transaction costs. Moreover, an investor that builds a portfolio based on trading volume and combine highly and lowly traded assets within a industry into portfolios, he or she can adjust positions based solely on movements in the most frequently traded assets. In summary, the most frequently traded assets leads the market while more thinly traded assets copy these movement with a lag. However, thin trading implies spurious cross-autocorrelation. Hence, the lead and lag results for thinly traded assets may turn spurious. However, for more frequent traded assets the lead and lag structure may still be valid.

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[25] A negative dependence story can be found in Poberta and Summers, 1988, and Fama and French, 1988 and a positive dependence story can be found in Taylor, 1986/2000.

[26] The positive autocorrelation results seem to disappear in the 1987-94 sub-period. Hence, we find no obvious autocorrelation results after the crash in 1987 for the thinly Norwegian market.

As the "in-Mean" coefficients are insignificant for all six series and we reject the residual risk hypothesis (Lehmann, 1990) for the Norwegian market. The degree of freedom coefficient (u_i) is strongly significant and induces deviation from normally distributed return series. The results seem therefore to indicate leptokurtosis in all six series independently of frequency of trading and non-trading effects.

The conditional variance equation report several interesting features from the thinly traded Norwegian market. Firstly, the ARCH-coefficient (shock) increases the higher the trading frequency. Hence, past squared errors influence strongest today's volatility for the most frequently traded series. The two market indices show results close to the two most frequently traded series. The past squared error for thinly traded series show a low volatility influence relatively to more frequently traded series. However, also for the conditional volatility we may find spurious past squared error results. In case of non-trading the observed return is zero and may produce artificial shocks for the volatility process. For thinly traded series we may therefore observe a spurious and too low ARCH coefficients. Secondly, the past conditional volatility influences strongest today's volatility for the thinnest traded series. Autocorrelation in the conditional volatility process seems therefore to be highest for the thinnest traded series. However, note that the non-synchronous trading and non-trading effects may cause spurious autocorrelation into the conditional volatility process. This may distort any volatility patterns for non-trading series.

Thirdly, the persistence ($a_i + b_i$) is strongest for the thinnest traded series. A clear picture of the persistence in the volatility process can be obtained by calculating the half-life of a shock to the process, that is, the time that it takes for half of the shock to have dissipated. Some algebra shows that the half-life in trading days for portfolio *i* may be calculated as[27] Half-life$_i$ = ln(0.5) / ln($a_{i,1}$ + $b_{i,1}$) and for calendar days as (252 $\cdot$ Half-life$_i$) / 365 = ln(0.5) / ln($a_{i,1}$ + $b_{i,1}$). Hence, Half-life$_i$ = (ln(0.5) / ln($a_{i,1}$ + $b_{i,1}$)) $\cdot$ (365/252). For our six return series we report the results for both formulas in Table 10. Table 10 suggests a significant difference in persistence length over the six series. Highest persistence is found for the thinly traded series, which report shock persistence for approximately 50 trading days. For the most frequently traded series the persistence is only 6.5 trading days. The information in the shock- and persistence-effects for series may be useful for investors building volatility strategies in an option market. One implication of an active option market that increases trading activity and therefore volume in the underlying asset may therefore be higher shock effects and lower persistence, that is a more erratic volatility. However, non-synchronous trading and non-trading effects may distort our result. Series strongly influenced by zero return observations may emphasise the autocorrelation in conditional volatility too much, which may result in spurious persistence coefficients. Applying results from the more frequently traded series imply rather strong non-trading effects. The three more frequently traded series plus the indices show all quite similar

[27] See Taylor, 1986/00 for details.

results. Hence, non-synchronous trading and non-trading effects may be severe for the conditional volatility in the thin Norwegian market. Fourthly, the constant ($m_{i,0}$) increases the higher the frequency of trading. Hence, as $m_{i,0} = \delta_{i,0} \cdot V_i$ and $\delta_{i,0} + a_{i,1} + b_{i,1} = 1$, the weight to the long-term average volatility seem to increase the higher the trading frequency. This feature implies that weight to the unconditional volatility is at its lowest for the most thinly traded series. Fifthly, asymmetric volatility (λ) is present in all series except for the most thinly traded series. The negative coefficients imply that it is the most frequently traded series that seem to show the highest asymmetry in the conditional volatility. The lack of asymmetry for the thinly traded series may also be attributed to the strong serial correlation. As both the weight to the long run average volatility and the shock effects is low in this series, the autocorrelation structure seem to be the dominant factor for the conditional volatility process. For all other series the asymmetric coefficient is negative and significant.

Turning now to the specification tests, we find several interesting features, which may originate from non-synchronous trading and non-trading effects. Firstly, the $Q^2(6)$ statistic report autocorrelation for the thinnest traded series. Neither market index nor more frequently traded asset series report autocorrelation in first and second moment residuals. Secondly, the ARCH test statistic reports conditional heteroscedasticity for the most thinly traded asset series. As for autocorrelation neither market index nor more frequently traded asset series report conditional heteroscedasticity. The RESET test statistic reports linearity in the conditional mean for all six series. Finally, the BDS test statistic reports general non-linearity for the thinnest traded series and the equal-weighted market index at some dimension (*m*). The three more frequently traded series and the vale-weighted index series show a BDS test statistic that fail to reject i.i.d. at any dimension (*m*). It seems therefore to be the case that the inclusion of the thinly traded series seems to introduce non-linearity into the equal-weighted market index. Hence, non-synchronous trading and non-trading effects cause non-linear dependence and model misspecification. Consequently, the ARMA-GARCH model specification seems not appropriate for thinly traded asset series.

The simple bias tests for volatility prediction show that especially bad news is not appropriately predicted in the GARCH-GJR model. However, only the most thinly traded series show biases when we perform a joint bias test. Moreover, the prediction bias is not strongly significant in the simple test statistic. When models are re-estimated for two sub-samples; 1983-1987 and 1988-1994, the second time period show a small change in autocorrelation for the conditional mean. In particular, after the crash in 1987 the slow adjustment process for the frequently traded series has changed to immediate adjustment. That is, no autocorrelation in the residuals for these series. The general conditional variance results are maintained in the sub-periods. However, later years (1988-1994) indicate an increased persistence in the variance process. For the thinnest traded series the conditional variance process show almost integrated GARCH.

Finally, Table 11 reports the first and second moments for the conditional volatility and the calculated unconditional volatility from the GARCH models[28]. The conditional volatility mean and the calculated unconditional volatility both report an U-shaped pattern as also reported for R_i^2 in Table 2. The calculated unconditional volatility is quite close to the conditional volatility mean. Moreover, the higher the trading frequency we find a strong and consistent increase in the standard deviation of the conditional volatility series. Hence, we find highest mean but lowest standard deviation for the conditional volatility process of the thinnest traded series and lower mean but highest standard deviation for the most frequently traded series. The results are in accordance with the ARCH/GARCH parameters for the conditional volatility estimations. Now calculation the index standard deviation divided by the mean. This index measures the relative uncertainty/change in the volatility process. The result shows clearly that it is the most frequently traded series and the two market indices that show highest changing volatility around a mean. Hence, non-trading effects show high volatility but lower changes in volatility. However, as the model specification is disputed for the most thinly traded series this result may be spurious. For option markets on individual and index series the estimations may produce valuable information for strategists. As the Norwegian market quote options for only continuously traded series, investors should be aware of this changing volatility result in applying the Black & Scholes option pricing formula. Estimates of the underlying asset's volatility may be very important for correct option pricing in these assets. To forecast future volatility using GARCH (1,1) model results is a well-known and easy exercise.

{INSERT TABLE 11 ABOUT HERE}

6 Summaries and Conclusions

We have modelled and estimated several ARMA-GARCH-in-Mean model specifications for the Norwegian thinly traded equity market. We apply trading volume as a proxy for trading frequency. As all the estimated ARMA-GARCH lag specifications for our series are BIC preferred, the model captures the autocorrelation and cross-autocorrelation structure in the conditional mean and the shocks, autocorrelation, persistence and asymmetry in the conditional volatility across varying trading frequencies. Moreover, the model measures the effect of "thick distribution tails" (leptokurtosis) through the degree of freedom parameter in the student-t distribution and potential residual risk is measured applying the in-Mean specification. The thinnest traded series and the equal-weighted index series report ARMA-GARCH lag structure misspecification. The results for these series may be spurious and must therefore be interpreted with great caution.

[28] The unconditional variance in a GARCH(1,1) specification is the long-run average volatility.

The study reports the following conclusions. ARMA-GARCH models seem to fit the Norwegian thinly traded market well, except for thinly traded series. The thinly traded series exhibit severe non-synchronous trading and non-trading effects, which induce data dependence and misspecification in the BIC efficient ARMA-GARCH model. The equal-weighted index series seem to inherit these data dependence and misspecification results in the Norwegian market. For relatively frequently traded series we find a consistent pattern in autocorrelation, cross-autocorrelation, volatility clustering and asymmetric volatility. For all these series we find insignificant specification test statistics. Hence, the ARMA-GARCH model and its parameter results for relatively frequently traded series suggest that previous regression models in the Norwegian market may have been wrongly specified owing to four specification failures. Firstly, a failure to efficiently incorporate the serial correlation structure in the conditional mean applying a BIC preferred lag specification. Secondly, a failure to incorporate the appropriate structure for measuring weight to long-run average volatility, shocks, autocorrelation, persistence and asymmetric volatility in the conditional variance equation applying a BIC preferred lag specification. Thirdly, a failure to specify thick-tailed distribution characteristics obtaining close to normally distributed residuals. Fourthly, a failure to control for data dependence in the model residuals, which implies spurious parameter results for thinly traded asset series.

Consequently, non-synchronous trading and non-trading seem to imply an extra challenge for modelling the dynamics in thinly traded markets. Classical regression models assuming conditional homoscedasticity seem obsolete. Moreover, for the applied ARMA-GARCH methodology, which uses the residuals for volatility specification, we find strongly significant misspecification for thinly traded series. Hence, the ARMA-GARCH methodology seems also to be a wrongly specified model in thin markets and thin series. As stochastic volatility models generate volatility processes independently of the conditional mean, the methodology may be an alternative model specification[29]. Alternatively, we may apply virtual returns (Campbell et al., 1997) and continuous time GARCH models (Drost and Nieman, 1993) for thin series. However, for relatively frequently traded series the ARMA-GARCH model seems appropriate.

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[29] See the SNP methodology of Gallant, Rossi and Tauchen, 1992.

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