Stock Return Volatility in Thinly Traded Markets

Abstract.

We study the mean and volatility of individual stocks in the thinly traded Norwegian equity market in an open (Monday-Friday) and closed (Weekends and Holidays) market. When the market is open we calculate mean and volatility ratios of consecutive trading versus 1, 2 and 3 days of non-trading. When the market is closed we calculate the mean and volatility of consecutive trading versus 1 (Holiday) and 2 (Weekend) days of non-trading. Building a model applying Brownian motions we can hypothesize mean and variance ratios in open and closed markets. The empirical results show that in an open market our hypothesized random walk mean and variance ratios are not rejected. Hence, in an open market the mean and volatility is independent of whether an asset is traded or not. In contrast, in a closed market our hypothesized variance ratios are strongly rejected. Hence, the hypothesized volatility is dependent on an open market and therefore volatility seems to be nearly unchanged in closed markets. The trading independence result together with Brownian motions implies that we should find near normal return distributions for both frequently and thinly traded assets. These findings prevail after adjusting for non-synchronous trading applying Poisson distributed trade arrivals and imply that trade arrivals and return distributions are independent.

Classification:

Keywords: non-synchronous trading and thin markets, mean and variance ratios, open and closed markets, trading and non-trading.

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1 Introduction

This paper studies mean and variance ratio for listed assets in the thinly traded Norwegian market. In the international finance literature much attention is focused on mean and variance processes and the information arrival process. Assuming trading volume and frequency of trading are able to proxy for information arrival and using the fact that the Norwegian market contains assets that show both thin and frequent trading, the Norwegian equity market may show trading characteristics that produce new knowledge to the microstructure of thinly traded markets.

Bachelier (1964) first developed the random walk model that uses a stochastic process called Brownian motions that assumes that security prices from transaction to transaction are independent, identically distributed random variables. Bachelier's model, together with the central limit theorem, suggests that price changes are normally distributed and that their variances will be linearly related to the time interval. However, we observe leptokurtosis in stock market return series. In the literature, one prominent explanation for the observed departure from Bachelier's model is the mixture of distributions hypothesis. This maintains that trade-to-trade asset returns exhibit leptokurtosis because they are really a combination of return distributions that are conditioned on information arrival. This means that periods of little or no information arrival result in observed return distributions different from periods when information arrives frequently. Hence, the return distributions on thinly traded stocks should differ from the distributions of stocks that are frequently traded. A thinly traded stock might not be traded for days, and when it is traded, it is often traded on low volume. Non-trading pricing processes are therefore an important factor for the understanding of return distributions. Moreover, pricing processes should be understood in both open (Monday through Friday) and closed markets (Weekends and Holidays). By studying mean and variance ratios for highly divergently traded stocks in an open market, the importance of trading activity may be measured. In addition, by studying the same ratios when the markets are closed we may find that pricing processes show independence of an open market. In combination these findings may produce results that imply new knowledge to the distribution assumptions that is the foundation of almost all-financial theory. Moreover, to my knowledge a simultaneous comparative study of open and closed markets, and trading and non-trading, has so far not been performed.

By developing and applying a model for non-synchronous trading we can hypothesize the mean and variance ratios for both open and closed markets. In open markets we hypothesize mean and variance ratios for consecutive days of trading versus 1, 2 and 3 days of non-trading. In closed markets we hypothesize ratios for consecutive days of trading versus 2 (Weekends) and 1 (Holidays) days of non-trading[i]. Hence, the ratios should produce characteristics in both open and closed markets. Our results imply that we are not able to reject the random walk model in an open market independently of trading frequency of individual assets. Hence, the mean and volatility is independent of the frequency of trading. Furthermore, in a closed market neither the hypothesized mean ratios can be rejected. However, in a closed market the hypothesized variance ratios are strongly rejected. Hence, the characteristics of the random walk model are not rejected in an open market but are strongly rejected in a closed market. The result opposes earlier classical results in international financial studies (see section 2 below).

As the Norwegian market is a competitive dealer market, the results from the study should be applicable to quite a number of thinly traded markets in Europe, America and Asia. Moreover, as this study analyses non-trading processes in the Norwegian computerized market, it should also be applicable to the US OTC (Over The Counter) market, which is both a computerized and by definition a thinly traded market.

The rest of the paper is organised as follows. Section 2 gives a literature review. Section 3 defines a model for the stochastic return process. Section 4 defines the empirical data. Section 5 reports our empirical results with economic implications and, finally Section 6 summarizes and concludes our findings.

2 Literature review

Empirical studies of mean returns and transaction arrivals typically reject simple assumptions as (1) mean returns are known to differ over weekends, holidays, and month of the year (French, 1980), Gibbons and Hess, 1981); (2) return variances are known to be lower during periods when the market is closed including weekends (Fama, 1965), exchange holidays (French and Roll, 1986) and overnight periods (Lockwood and Linn, 1990); (3) return variances exhibit season differences such as across days of the week (Lockwood and Linn, 1990) and near the open close of trading hours (Wood et al., 1985), Harris, 1986, and McInish et al., 1990). An increasing body of evidence following GARCH specifications indicates that return variances are also auto-regressive (French, Schwert, and Stambaugh, 1992; Solibakke, 1997).

Similarly, transaction arrivals do not appear to arrive independently over time. For example, Jain and Joh, 1988 find that trading frequency is dependent on the time of day in an open market; namely, trading is heavier in the beginning and end of the trading day and lighter in the middle. In a semi-non-parametric GARCH setting, Gallant, Rossi and Tauchen, 1992, find that return variances are serially, cross and serially cross dependent. That is, variance and trading volume are jointly determined both cross sectional and over time.

In order to investigate the nature of the returns of differently traded assets, a good starting point is the return generating model set forth in Scholes and Williams (1976,1977). This formulation models returns and transaction arrivals and will be discussed fully in Section 3 below. Most studies in this literature review confirm that returns and volume are simultaneously and jointly determined and are linked to information arrival. If we can determine how returns and trades are distributed, the former literature should also give increased understanding of how information changes affect the market.

2.1 Returns in open and closed markets

When stock markets are closed, no trades occur in these markets. Thus although investor expectations about returns may have changed, or information arrives that would alter the expected return on a stock, price effects are not observable until the markets reopen. A common problem, therefore, in theoretical and empirical studies of financial markets is the identification of returns when the markets are closed or in non-trading periods. Many theoretical models of the return generating process assume that price changes are independent of when and how often trades occur. That is, there exists a "true" price whether or not a trade occurs. Thus, under this proposition a return is generated both over weekends and evenings when the market is closed and in a thin market when the market is open but the asset is not frequently traded (e.g. Scholes & Williams (1976, 1977 and Lo & MacKinlay (1990)). This assumption draws theoretical justification from models of symmetrically informed traders from, for example, Marshall (1974) and Rubinstein (1975) in which prices can change without trading as investors' expectations change in unison.

An opposing hypothesis is that returns and transactions occur only when information arrives. With regard to returns, Ross (1989) assumes that information arrives through a Martingale process, and though no-arbitrage conditions, demonstrates that return variance is directly related to the flow of information. Similarly, transaction arrivals are also likely related to the flow of information. In this case, price changes when there is new information and the price changes are coincident with trades. Non-trading periods could represent periods in which no information arrives and hence price and return do not change.

The true relationships between information arrivals, transaction frequency, and return, lie probably somewhere between these two extremes. For instance, French and Roll (1986) find that prices are more volatile when markets are open than when they are closed. Their results suggest that there is a continuous component to the return, as well as a component that is driven by the information arrival. If one assumes further that information arrival is more likely to happen when markets are open, then one is likely to find that trading frequency is positively related to the mean and variance of returns.

Booth and Chowdhury (1996) confirm that stock return variances are higher during trading hours than during non-trading hours, and provide evidence consistent with the private and public information hypothesis and against the noise trading explanations. Subrahmanyan (1991) shows that when informed traders are risk averse, noise trading raises price volatility because these traders respond less aggressively to an increase in noise trading than do risk-neutral informed traders. Further, De long et al. (1990) suggest that presence of a certain type of noise traders, “positive feedback traders”, may lead to an increase in volatility. This occurs when informed speculators, rather than taking positions opposite the positive feedback traders, reinforce the market price movement away form its fundamental value.

2.2 Returns of individual stocks in Thinly Traded Markets

In a theoretical development of the role of thinness in securities markets, Cohen et al. (1978) use compound Poisson processes to model the discrete time arrival of transactions. They show that under heterogeneous expectations, variance is inversely related to the market value of a stock. Using total market value as an inverse proxy for thinness, they find that thinness is a significant determinant of variance. Silber (1975) investigates empirically the effect of thinness on stocks listed on the Tel Aviv Stock Exchange. He finds that two salient characteristics of thinness are a large bid-ask spread and a large variability in price per unit of excess demand. Moreover, he examines the relationship between price change volatility and the following variables: (1) the volume traded of each security; (2) the total supply outstanding of each security; (3) the number of stockholders; (4) the total asset of the firm; and (5) the number of days on which no trading occurred in each security during a particular interval. His results show that trading volume is the best indicator of lack of thinness for equity markets. In the bond market, the number of days of non-trading is the most consistent indicator of thinness followed by trading volume and size of the issue.

Moreover, because securities in thin markets often trade only once every several days, there exists a measurement problem for empirical studies that use daily returns. Observed trade-to-trade does not correspond to true daily returns since securities do not trade every day at market close. Therefore, any use of reported daily returns rather than true returns results in the econometric problem of errors in variables. Non-synchronous trading really means that asset prices are recorded at time intervals of one length when in fact they are recorded at time intervals of other, possible irregular, lengths. As shown by Scholes and Williams (1977) and Lo and MacKinlay (1990), failing to account for non-synchronous trading problem results in overstated variances and spurious auto- and cross-correlation. Moreover, Scholes and Williams (1977) find those ordinary least squared estimators for alphas and betas in the market model are biased and inconsistent.

Lo and MacKinlay (1990a) develop a stochastic model of non-synchronous asset prices that accommodates the problem of non-trading. In particular, their model assumes more realistically that the time between trades is stochastic rather than limiting the model by forcing a trade per day, as Scholes and Williams did (1977). Lo and MacKinlay also derive closed form expressions for the unconditional means, variances and covariance of observed returns as functions of the non-trading process. Among other results, they find that non-synchronous trading does not affect the means of individual returns, while; on the other hand, it increases the observed variance of these same security returns (if mean returns are different from zero)[ii].

3 A Model for a Stochastic Return generating process

Assume that the return is a continuously compounded rate R_t per trading period $\{t\text{-}1, t\}$. The price of the asset at time t is log-normally distributed and denoted P_t, so $R_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$ and normally distributed. We therefore assume further that P_t is characterized by a Brownian motion with parameters μ and σ^2 selected so that $E(R_t) = \mu$ and $Var(R_t) = \sigma^2$. The simplest representation of the (arithmetic) Brownian motion (Bachelier, 1964) is $R_t = \mu\, dt + \sigma\, dz_t$, where dz_t is the increment of a Wiener process, defined as $dz_t = \varepsilon_t \cdot \sqrt{dt}$, where ε_t has zero mean and unit standard deviation, $E(dz_t) = 0$ and $Var(dz_t) = E((dt)^2) = dt$. μ is called the drift parameter, and σ^2 the variance parameter. Note that over any time interval dt, R_t, is normally distributed, and has expected value $E(R_t) = \mu \cdot dt$ and Variance $\mathrm{Var}(R_t) = \sigma^2 \cdot dt$. Furthermore, note that a Wiener process has no time derivative in a conventional sense; $\frac{dz_t}{dt} = \varepsilon_t \cdot (dt)^{-1/2}$, which become infinite as dt approaches zero.

In general, the return $R_{a,b}$ over a time-period $\{a,b\}$ is given by $R_{a,b} = \ln\left(\frac{P_b}{P_a}\right)$, where P_a and P_b is the observed prices at time a and b ($a < b$). When a and b is constantly changing among the component stocks, this may cause non-synchronous trading, as the time interval between observations change (possibly irregular). If we assume that the return generating process $R_{a,b}$ follows an arithmetic Brownian motion a model for $R_{a,b}$ is given by

$$R_t = (b - a) \cdot \mu + \sigma \cdot (dz_b - dz_a) \qquad (1)$$

where $dz_b - dz_a$ is normal with mean 0 and variance $b - a$. Now, let $t\text{-}1\text{+}v$ be the time of the last trade during trading period $\{t\text{-}1, t\}$ and let k be the number ($k = 0,1,2,3$) of trading periods after $\{t\text{-}1, t\}$ in which there are no trades ($v > 0$). This time-definition gives a definition of v that is the time until the last trade of day t. This means that there is at least one trade during $\{t - 1 + k + 1, t - 1 + k + 2\}$. Let u be the time for the trading period $\{t + k, t + k + 1\}$. I illustrate the notation and trading sequence in Figure 1.

{Insert Figure 1 about here}

R^{obs} represents the observed $k + 1$ period return based on the last trades in period $\{t\text{-}1, t\}$ and $\{t + k, t + k + 1\}$. Assuming that k, and v are independent and that u and v are identically, independently distributed, employing equation (1) leads to

$$R^{obs} = \mu \cdot ((t + k + u) - (t - 1 + v)) + \sigma \cdot (dz_{(t+k+u)} - dz_{(t-1+v)}) \text{ and}$$

$$R^{obs} = \mu \cdot (k + 1 + u - v) + \sigma \cdot (dz_{(t+k+u)} - dz_{(t-1+v)})$$

Using standard result from stochastic calculus gives us that the expected return $E(R^{obs})$ for a given day equals

$$E(R^{obs}) = \mu \cdot ((k + 1 + E(u) - E(v)) + \sigma \cdot (E(dz_{(t+k+u)} | k) - E(dz_{(t-1+v)} | k))$$

$$E(R^{obs}) = \mu \cdot (k + 1) + \sigma \cdot (k + 1) \cdot (0)$$

$$E(R^{obs}) = \mu \cdot (k + 1).$$

That is, the expectation of observed returns is equal to the true mean one-period return multiplied by $k + 1$. This is consistent with both Scholes and Williams (1976, 1977) and Lo and Mac-Kinlay (1990) who find that mean returns are unaffected by non-synchronous trading. To compute the observed variance for R^{obs}, $Var(R^{obs} | k)$, we can use the known fact that $Var(A) = Var(E(A | B)) + E(Var(A | B))$, for arbitrary A and B. Thus,

$$Var(R^{obs} | k) = Var(E(R^{obs} | k, u, v)) + E(Var(R^{obs} | k, u, v)).$$

Some algebra leads to:

$$Var(R^{obs} | k) = Var(\mu (k + 1 + u - v)) + E(\sigma^2 (k + 1 + u - v)) \text{ and}$$

$$Var(R^{obs} | k) = \mu^2 \cdot Var(0 + u - v)) + \sigma^2 \cdot E(k + 1 + u - v).$$

Now, since u and v are identically and independently distributed

$$Var(R^{obs} | k) = 2 \cdot \mu^2 \cdot Var(u) + \sigma^2 \cdot (k + 1) \tag{2}$$

which provides a general form of the relationship between observed and true variance. Equation (2) allows for correction of the variance of measured returns in thinly traded markets. If we determine the variance of the measured returns, their means, and the variance of the time interval between the beginning of a day and the last trade, then the true variance of returns can also be determined. However, unless we assume a specific distribution for the trading process, *Var* (u) cannot easily be determined and a closed form relationship between measured and actual returns cannot be obtained. Section 5 shows the adjustments of the variance when we assume a Poisson distribution of the trading process.

4 Data

The study uses daily and return series for Norwegian stocks spanning the period from October 1983 to February 1994. This high frequency time series database gives at most 2611 observations for each asset. However, the Norwegian sample was chosen as this thinly traded equity market has sufficient number of divergently traded stocks (long periods of trading and non-trading) that make it possible to study pricing processes during an open and closed market.

Daily trading volume data was obtained from the 'Oslo Børs Informasjon' Database. All stocks in the database are used in the analysis. We have divided the period into two sub-periods; one period from October 1983 to September 1987 (1019 observations) and one period from December 1987 to February 1994 (1546 observations). Daily stock returns are calculated as the change in the logarithm of successive closing prices. Sample assets satisfy the following criteria:

(1) The assets are listed at the Norwegian Stock Exchange and information of daily asks, bids and settlement prices including trading volume were available.

(2) The assets must have at least 5 return observations of both consecutive ($k=0$) and k_O (1, 2, 3) non-trading day(s) when the market is open and k_C (Weekend and Holiday) when the market is closed.

Specifically, if an asset is selected for the $k_O = 1$ sample[iii], then the asset has registered at least 5 consecutive daily trading observations and 5 *one* day non-trading observations. For the weekend sample k_C, the asset must have registered at least 5 consecutive daily trading observations and 5 *Monday* return observations. Finally, individual asset returns for other k_O's (more than 3 days of non-trading in an open market) and k_C's (several consecutive holidays), are discarded mainly owing to few observations[iv].

Table 1 presents summary data on frequency of trading/non-trading for the stocks in all five above defined samples. As can be seen from Table 1, not all the stocks were listed for the entire period and sub-periods. In order to determine the percentage of trading days for a given stock, only days on which the stock is listed and a first settlement price are quoted, are included in the calculations. Among the stocks in the samples for $k_O = 1, 2, 3$, there is quite a large range in the frequency of trading. For $k_O = 1$ and the entire period, from June 1 1983 to February 1 1994, the mean percentage of trading days is 60.39%, while the minimum is 5.47% and the maximum is 99,25% (not reported). Hence, the $k_O = 1$ sample contains both frequently and infrequently traded stocks. Moreover, for weekends and holidays k_C, the percentage of trading days increases in the sample because these samples also contain the most frequently traded assets[v]. Finally, we divide the periods as shown above due to two market observations. The first is the crash in October 1987 and the second is the fact that Oslo Stock Exchange switched to an electronic trading system early in 1988.

Table 1 implies that return observations on consecutive days are more numerous than *k*-days' returns that include *k* non-trading day(s). Consecutive-day variances are therefore in general, estimated with more precision than variances measured over $k + 1$ days. The total sample contains 227 assets spanning the whole period. For $k_O = 1, 2, 3$, *and* $k_C = 1$*(holidays) and* 2 *(weekends)* we emerge with 220, 161, 107, 173 and 224 individual assets, respectively. These five samples of individual asset for return and variance calculations are used in the remaining sections to test our hypothesis.

{ Insert Table 1 about here. }

5 Empirical Results

5.1 Means and Variances

To calculate means and variances for our observed returns, we apply the following procedures. Firstly, each asset's compounded mean returns are calculated for $k_O = 0, 1, 2$ *and* 3 non-trading days[vi] in an open market and for weekends $(k_C = 2)$ and holidays $(k_C = 1)$ in a closed market. Figure 2 illustrates these calculations for consecutive mean returns $(k = 0)$ and for three day mean returns $(k = 2)$. Hence, for $k=0$, the return is the calculated continuously compounded return using closing prices at $t = 1$ and $t = 2$. With two days of non-trading also shown in Figure 2, the 3-day mean returns is calculated as the continuously compounded return over 3 days using closing prices at $t = 2$ and $t = 5$.

{ Insert Figure 2 about here. }

From these mean returns, sample means and variances are calculated at the sample level for each category $(k=0, k_O=1,2,3$, and $k_C=2$ (weekends) and $k_C=1$ (holidays). Hence, each asset will have a mean and variance of returns for consecutive days of trading as well as for periods in which there are $k_O = 1, 2,$ *and* 3 days of non-trading in an open market. Moreover, we calculate mean and variance for periods in which there are $kC=1$ *(holidays) and* $kC = 2$ *(weekends)* days of non-trading when the market is closed. Note especially that because an asset must have 5 observations of each return class, our procedures imply that frequently traded assets which appeared in the $k = 0$ versus $k_O = 1$ comparison were not likely to appear in the $k = 0$ versus $k_O = 3$ comparison. Hence, we account for the rapid decline of assets for samples $k_O = 1, 2$ *and* 3. Formally, the grand means and variances for a given *k* and all sub-periods are calculated as

$$E(R^{Obs} \mid k) = \frac{1}{M_k} \cdot \left[\sum_{i=1}^{M_k} \left(\frac{1}{N_k} \cdot \left(\sum_{j=1}^{N_k} R_{ij}^{Obs} \right) \right) \right] \tag{3}$$

and

$$Var(R^{Obs} \mid k) = \frac{1}{M_k} \cdot \left[\sum_{i=1}^{M_k} \left(\frac{1}{N_k} \cdot \sum_{j=1}^{N_k} \left(R_{ij}^{Obs} - E\left[R^{Obs} \mid k \right] \right)^2 \right) \right] \tag{4}$$

where R_{ij}^{Obs} is the total returns for asset *j* period *i* ; N_k is the total number of observations for a given asset *j* and non-trading period *k*; M_k is the total number of assets in the sample for *k* non-trading periods. Thus for each time period we produce sample mean and variance across assets for consecutive days of trading and for $k_O = 1, 2$ *and 3* days of non-trading in an open market and for $k_C=2$ (weekends) and $k_C=1$ (holidays) in a closed market.

If the return generating process is, as is commonly assumed, arithmetic Brownian motion, then stock return means and variances are linearly related to the time interval. Hence, the mean and variance of returns over a period in which there is no trading for *k* days should be *k* times the mean and variance of consecutive days of trading. Table 2 reports the mean returns for each non-trading period for both an open and closed market. The null hypothesis is that the non-trading day mean return $E(R^{obs} \mid k)$ should be $k + 1$ times the consecutive day returns. The alternative is that the non-trading day mean return $E(R^{obs} \mid k)$ is different from $k + 1$ times the consecutive day returns. Therefore our first hypothesis becomes

Hypothesis 1:

H_0 : $E(R^{obs} \mid k_{O/C}) = (k_{O/C} +1)\, \mu$, for k_O = 1, 2, 3, and k_C = 1, 2,

H_A : $E(R^{obs} \mid k_{O/C}) \neq (k_{O/C} +1)\, \mu$, for k_O = 1, 2, 3, and k_C = 1, 2.

When the market is open applying a two tail t-test[vii], we find the probability that |*t*| takes a value higher than the calculated value for the degrees of freedom is high (>90%). This suggests that when the market is open, the null hypothesis of $(k_O+1)\mu$ mean return is not rejected for any k_O non-trading days. That is, the mean returns may in fact be an integer multiple of the time interval. The result is consistent for all non-trading days when the market is open over all three periods. However, note the close to zero and negative returns for the non-trading days in all periods in contrast to positive returns for consecutive days of trading. When the market is closed applying the same two tail t-test, for weekends $(k_C=2)$ and holidays $(k_C=1)$ we find the same results. That is, one (two) day(s) of non-trading returns when the market is closed may in fact be equal to one (two) days of consecutive day returns. The results are consistent over all three periods.

{ Insert Table 2 about here. }

The first null hypothesis is strongly rejected and is probably driven by the fact that variances are large relative to the magnitude of mean returns. Therefore, to extend and further analyse the returns, a second hypothesis is tested. Does a mean *k + 1* day return equal zero? This hypothesis is set out below.

Hypothesis 2:

H_0 : $E(R^{obs} \mid k_{O/C}) = 0,$ for $k_O = 1,2,3$, and $k_C = 1, 2$,

H_A : $E(R^{obs} \mid k_{O/C}) \neq 0,$ for $k_O = 1,2,3$, and $k_C = 1, 2$.

In all cases when the market is both open and closed, using the same form of t-test as above, the null hypothesis is not rejected. That is, the mean returns for all categories may in fact be zero. As above, this result is also consistent over all three periods. This result is also probably driven by a large variance relative to mean. Finally a third hypothesis is tested. Does mean *k + 1* day-return equal mean consecutive days return? This hypothesis is also set out below.

Hypothesis 3:

H_0 : $E(R^{obs} \mid k_{O/C}) = \mu,$ for $k_O = 1, 2, 3$, and $k_C = 1, 2$,

H_A : $E(R^{obs} \mid k_{O/C}) \neq \mu,$ for $k_O = 1, 2, 3$, and $k_C = 1, 2$.

For all cases when the market is both open and closed, the null hypothesis is not rejected. That is, the mean returns may in fact be equal to consecutive day mean returns. All three hypotheses are therefore not rejected. The result support Scholes and Williams (1977) and Lo and MacKinlay (1990c) mean results. Non-trading when the market is open and closed does not significantly diverge from the consecutive day mean returns.

However, some observations are interesting and readily available from Table 2. For the non-trading cases when the market is open, we find consistent positive mean consecutive day returns. The returns are remarkably stable showing results of about 0.3% to 0.4% for all the three sample periods. The consecutive daily returns are for all three periods lowest for $k_O = 1$ and highest for $k_O = 3$. This suggests that frequently non-traded assets show positive returns when they are traded for consecutive days. A possible interpretation is that lowly traded assets (periods of non-trading) are rewarded with a highly daily return when they are traded for consecutive days. Hence, for thinly traded assets a possible interpretation of our results is that there is a trading effect in the market. The positive trading return effect seems to increase the thinner the asset is traded in the market. Moreover, the non-trading periods k_O mean returns are mainly negative except for $k_O = 1$ and *2* in the first sub-period 1983-87. However, the returns are close to zero. Therefore, our results suggest that assets experience higher negative returns the longer the non-trading periods. In this case a possible interpretation is that there is a non-trading effect for long non-trading periods. The non-trading negative return effect seems to increase strongly from $k_O = 1$ to $k_O = 2$ *and 3*. Trading volume in form of the number of trading and/or non-trading days is therefore a candidate for an independent variable in cross-sectional regressions of daily stock returns[viii]. Hence, in summary, the result of the sample means suggest positive consecutive trading and negative non-trading mean return effects in the market. Moreover, it seems that the longer the non-trading period the higher the negative market means return effect.

When the market is closed, returns following holidays ($k_C = 1$) seem to show consistently high positive returns for all the three time periods. Moreover, the weekend effect ($k_C = 2$) seem to have moved from a positive anomaly to a negative but close to zero non-anomaly after 1987. Hence, our results seem to suggest a shift away from the well-known weekend anomaly effect. For the sub-period 1983-87 I find a high positive weekend effect well beyond other weekdays average consecutive trading day returns. The same result dominates the whole period 1983-94. However, studying the period 1987-94 we find a negative and close to zero average weekends return. This average return is below the other days' average consecutive trading day returns. The well known weekend or Monday effect seems to have moved from a positive before the crash to a negative and close to zero return after the crash.

Table 3 reports the main results for the grand return variances. In the right most columns of Table 1, the average variance ratio for each sample period and non-trading duration k are presented. Variance ratios are determined by dividing variance measured over $k+1$ non-trading days for $k_O = 1,2,3$ in an open market and $k_C = 2$ (weekends) and $k_C = 1$ (holidays) in a closed market, by the variance of consecutive day $(k = 0)$ returns[ix]. If returns follow a random walk, the variance ratios of each trading day category k should equal $k + 1$. This hypothesis is set out below.

Hypothesis 4:

H_0 : $$\frac{\mathrm{Var}(\mathrm{Robs} \mid \mathrm{k}_{O/C})}{\mathrm{Var}(\mathrm{Robs} \mid \mathrm{k} = 0)} = k+1,$$ for $k_O = 1, 2, 3$, and $k_C = 1, 2$,

H_A : $$\frac{\mathrm{Var}(\mathrm{Robs} \mid \mathrm{k}_{O/C})}{\mathrm{Var}(\mathrm{Robs} \mid \mathrm{k} = 0)} \neq k+1,$$ for $k_O = 1, 2, 3$, and $k_C = 1, 2$.

The conventional method of using the F-test[x] for testing variances of samples is to take their ratio, adjust for degrees of freedom and compare this to one. In our case to perform the F-test in this fashion, one must first multiply Var_0 by $k+1$ or divide Var_{k+1} by $k + 1$ and then proceed with the conventional method. For example, a variance ratio based on two-day returns with $k = 1$ day of non-trading should equal two.

{Insert Table 3 about here.}

Employing an F-test for each period and for each non-trading category k, the null hypothesis at 5% is shown by a * to the right of each k for all five samples and sub-periods. Table 3 show that for all three periods and all non-trading periods $(k=1,2,3)$ in an open market, the random walk hypothesis is not rejected. Variances for periods that include one or more k-days of non-trading appear to be equal to the prediction of the random walk model. This suggests that the return variances are created both on trading and non-trading days[xi] in an open market. Consider the variance ratio over the entire sample period for $k_O = 1$, that is 2.189. If return variance on the first day of trade after a one-day non-trading period is equal to consecutive day variances, then the non-trading day variance is 109.5% of the variance over 1 consecutive trading days. The same number for a two-day (three-day) non-trading period is 85.12% (108.3%) variance of a 2 (3) days period of consecutive trading.

To consider the situation when the market is closed we study the weekend and holiday mean results. The k_C-days will here be days when the market is closed. For all three periods and for both weekends ($k_C=2$) and holidays ($k_C=1$), the random walk hypothesis is strongly rejected. Variances on periods that include 1 or 2 days when the market is closed do not follow the prediction of the random walk model. This suggests that the return variances be created only when the market is open regardless of trading or non-trading periods.

We now want to explore the conjecture that the return variances are created primarily on days when active trading takes place in an open market. Such a test is performed by using the hypothesis that the variances of consecutive days of trading when the market is open, are equal to the variances of those periods that include non-trading days when the market is open ($k_O=1,2,3$) and closed ($k_C=2$ (weekends) and $k_C=1$ (holidays)). This is equivalent to testing the hypothesis that the ratio of variances is one:
Hypothesis 5:

$$H_0: \quad \frac{\mathrm{Var}(\mathrm{Robs} \mid k_{O/C})}{\mathrm{Var}(\mathrm{Robs} \mid k = 0)} = 1, \qquad \text{for } k_O = 1, 2, 3, \text{ and } k_C = 1, 2,$$

$$H_A: \quad \frac{\mathrm{Var}(\mathrm{Robs} \mid k_{O/C})}{\mathrm{Var}(\mathrm{Robs} \mid k = 0)} \neq 1, \qquad \text{for } k_O = 1, 2, 3, \text{ and } k_C = 1, 2.$$

Table 3 shows that this null hypothesis is rejected over all three periods for non-trading days in an open market. In contrast, the null hypothesis is not rejected over the two non-trading days in a closed market. That is, for weekends and holidays when the market is closed the variance is constant; for non-trading days when the market is open, the variance is $k_O + 1$ the variance of consecutive days of trading. Hence, return variances show activity in an open market regardless of trading and non-trading.

Our results cannot reject the Lo and MacKinlay (1990) proposition that the variance increases during non-trading. However, the increase is relative to consecutive day variance ratios. When the market is open the variance ratio is proportional to k_O number of non-trading days. Hence, the variance is independent of trading or non-trading but requires an open market. The random walk model and Brownian motion of asset returns is therefore independent on trading volume in an open market. These results suggest that the return generating process is enduring and constant as long as the market is open. In contrast, during weekends and holidays (k_C) when the market is closed, the return generating process halts. Hence, the hypothesis of Scholes and Williams (1977) and Lo and MacKinlay (1990) is not rejected when the market is open, while the opposing hypothesis in Ross (1989) is not rejected when the market is closed. As a consequence, the results imply that information is immediately assimilated in an open market. When the market is closed no information is assimilated and an accumulation takes place. This accumulated information may explain the extra volatility at open found by McInish and Wood (1990). Moreover, the extra volatility at close also found by McInish and Wood (1990) may be explained by approaching a period of no information assimilation. Overnight the information is accumulated, which imply increased volatility at open.

Our results also imply that changes in the information flow do not influence the pricing processes in the market over time. It is the market mechanism itself that seems to affect the pricing process. Therefore, a continuously open market may imply a constant time proportional variance for individual assets without start and stop of variance processes. For individual investors the results imply that in a closed market, no price processes are at work. However, the information flow does not stop and will accumulate in closed markets. Some extra attention from investors is therefore warranted at open and close of the markets.

5.2 Variances - Adjusted for Non-synchronous trading

Our results above suggest that any adjustment to the variance shown in (2), should *not* influence our findings. As shown in section 3 above, a test of this proposition is to assume that the occurrences of trades follow a Poisson distribution. This means that trades occur as a Poisson process with parameter λ, where λ is the mean number of trades per period. Let $s = 1 - u$ represent the time remaining in a given trading period after the last trade. Then s is distributed exponentially $\lambda e^{-\lambda s}$ $(\lambda > 0)$ on $0 \leq s < 1$ with the probability of no trade during any trading day of $PROB\ (s \geq 1) = \int_1^{\infty} \lambda \cdot e^{-\lambda \cdot s} ds = e^{-\lambda}$. If s is conditioned on at least one trade per trading day, the density function is $f(s) = \dfrac{\lambda \cdot e^{-\lambda \cdot s}}{1 - e^{-\lambda}}$, $0 \leq s < 1$. Given the density function $f(s)$ above, and remembering that $u = 1 - s$, the conditional variance of u is calculated using integration by parts:

$$Var(u) = \frac{\lambda^2 \cdot e^{\lambda} + e^{(-\lambda)} - 3 + 3 \cdot e^{\lambda} - \lambda^2 - e^{(2\cdot\lambda)}}{\lambda^2 \cdot \left(e^{\lambda} - 1\right)^2 \cdot (-1 + e^{-\lambda})} \tag{5}$$

Substituting equation (5) into equation (2) above provides the relationship between the observed variance and the true variance, given that the process describing trades is Poisson. Therefore,

$$Var(R^{obs} \mid k) = 2 \cdot \mu^2 \cdot \left[\frac{\lambda^2 \cdot e^{\lambda} + e^{(-\lambda)} - 3 + 3 \cdot e^{\lambda} - \lambda^2 - e^{(2 \cdot \lambda)}}{\lambda^2 \cdot \left(e^{\lambda} - 1\right)^2 \cdot \left(-1 + e^{(-\lambda)}\right)} \right] + \sigma^2 \cdot (k+1) \,. \quad (6)$$

All the inputs necessary to relate the measured variance to the true variance are now at hand. If an empirical estimate is used for λ, the mean arrival rate of transactions, then this model provides an approximation of how much of the observed variance due to non-synchronous trading and provides a means to correct measured variances for non-synchronous trading. Equation (6) captures the general form of the relationship between observed and true variance. As in Scholes and Williams case, the observed variance is dependent on the mean return and always overstates the true variance of returns. Increasing time between trades increases the observed variance. And as λ increases, the observed variance quickly approaches the true variance; in other words, as trading becomes more frequent, measurement problems diminish. This is illustrated in Figure 3.

{Insert Figure 3 about here.}

To estimate λ, the mean number of trades per period, we take the number of trading days (days on which trading volume > 0) and divide by the total number of possible trading days. If one trading day is taken to be one period, this provides an approximation of the empirical probability density of trading for one day. This estimate can then be used to estimate λ. The Poisson process is given by $P(X = x) = \left(\frac{\lambda^x}{x!} \right) \cdot e^{-\lambda}$ where *x* = the number of trades, and λ = mean number of trades per period. The probability of no trading is $P(X = 0) = \left(\frac{\lambda^0}{0!} \right) \cdot e^{-\lambda} = e^{-\lambda}$. Therefore,

$$\lambda = -\ln\left(P\left(X = 0\right)\right) . \quad (7)$$

For *P (X = 0)* we substitute the percentage number of days in one year on which there are no trades. Now using equation (5) and (6) the true variance σ^2 can be estimated because all other variables are known empirically. Using observed means and variances from the results above and estimated λ*'s* using equation (7), estimates of true variances are calculated according to equation (5) and (6) and presented in Table 4. Rearranging equation (6); $\sigma^2 = \frac{Var(R^{Obs} \mid k) - 2 \cdot \mu^2 \cdot [Var(u)]}{k+1}$.

As the mean transaction arrival rate increases, the variance of *u*, the time between the beginning of the day and the last trade is reduced. Given *k* constant and $\mu_i = \mu \neq 0$ for all assets *i*, more thinly traded stocks will have bigger adjustments to measured variances. And, for a particular μ_i and λ , as the length of the trading period increases, the correction to measured variances diminishes at a rate $\frac{1}{k+1}$.

In Table 4, the significance levels in Table 3 prevail for each variance ratio test after adjusting for non-synchronous trading and non-trading when the market is open and closed. In fact, the magnitudes of the adjustments for non-synchronous trading do not materially affect the comparisons. Non-synchronous trading alone is unable to explain the difference in return variance between trading and non-trading periods when the market is open and closed. These results therefore confirm our findings in Section 5.1, which implied that non-synchronous trading should not affect our results.

{Insert Table 4 about here.}

6 Summary and Conclusions

The main result of this study indicates that return variances are related to whether or not the market is open. In particular, return variances over non-trading periods from 1 to 3 days in an open market appear to be equal to *k+1* the return variances over consecutive periods of trading as implied by the random walk model. However, return variances over non-trading periods of 1 to 2 days in a closed market appear to be equal to the return variances over one consecutive day of trading. Hence, in all three sub-periods, (1) variances for all non-trading periods in an open market do conform to the random walk model and (2) variances for all non-trading periods when the market is closed do not conform to the random walk model. The economic interpretation is that even though we assume that the information flow is almost constant, it is the market mechanism that influence and directs the pricing processes. Hence, a continuous open market may produce a constant proportional variance for individual assets irrespective of trading intensity and non-trading.

The model of non-synchronous trading was also developed to allow for correction of the measurement error inherent in periods of infrequent trading. We assume a Poisson distributed trade arrival process. Consistent with other findings (Scholes & Williams 1976,1977 and Lo and MacKinlay, 1990), the model analytically shows that while observed mean returns are unbiased, observed variances consistently overstate true variances. Hence, our results imply that the longer the non-trading period the lower the measurement error for variance calculations. Despite the correction for non-synchronous trading, the results remain materially unchanged. With reference to our first findings, this result was to be expected.

The paper may therefore conclude that variances are not affected of trading or non-trading processes in an open market. However, when markets are closed the variances is nearly unchanged. A continuous open market (24 hours) may therefore produce a constant variance parameter in the random walk model, that is, a time proportional volatility.

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[i] Weekend returns are measured from Friday close to Monday close. Hence, 3 days return with 2 days of non-trading in a closed market. Holiday returns are measured from close on the day before one-day holiday to close on the day after one-day holiday. Hence, 2 days return with 1 day of non-trading.

[ii] Lo and MacKinlay used the returns of ten size-sorted portfolios for daily, weekly and monthly data from 1962 to 1984. The study confirms the earlier results in Lo and Mackinlay (1990c). Using portfolio returns imply that the study is mainly a correlation study. Our study is slightly different from Lo and MacKinlay's. Firstly, we employ individual stocks. This makes mean and variance characteristics for divergently traded stocks possible as observed trading volume is available. Secondly, for each asset in the sample we calculate the multiple-day returns and variances including one or more non-trading days. For each asset in the sample these multiple returns and variances will be compared to returns and variances measured over consecutive trading days given that the number of non-trading days in each multiple-day return observation is known. Finally, sample averages of returns and variances are calculated. These numbers are used to calculate ratios. Consequently, the relation of measured returns and variances to true returns and variances in the presence of non-synchronous trading and non-trading when the market is open and closed conditional on a known number of non-trading periods must be worked out. The random walk model with normal i.i.d. increments is used as starting point.

[iii] k_O non-trading days in an open market; k_C is non-trading days in a closed market.

[iv] This feature will also exclude holidays on Monday and Friday in the samples.

[v] Excluding the most frequently traded assets from the weekend and holiday samples does not materially change our finding.

[vi] We associate non-trading with zero trading volume, *k=0* means consecutive days of trading. Non-trading periods lower than 3 days implies that only observation within the 5 weekdays is accounted for in the calculations.

[vii] The t-statistic use: t = $\left|\frac{(R_k / (k+1)) - \mu}{\sigma}\right|$, where R_k is k non-trading days return, *m* is the consecutive days of trading return, σ is the standard deviation, *k* is the number of non-

trading days. The statistic gives the probability that |*t*| takes a value greater than the calculated value for the stated degrees of freedom. This is thus a two-tailed test.

[viii] An non-parametric integrated hazard function is therefore of considerable interest.

[ix] Note that all variance ratio calculations are done within the same asset and the numbers we report are the average over all the assets in the sample.

[x] The F-test use: $\frac{\sigma_1^2 / f_1}{\sigma_2^2 / f_2} =$, where $\sigma^2{}_1$ and $\sigma^2{}_2$ are independent χ^2 variables with f_1 and f_2 degrees of freedom respectively

[xi] Heinkel and Kraus (1988) suggest a model of non-trading stocks, which may fit the empirical data well. These authors assume that the return variance of individual stocks on days in which they do not trade is equal to the return variance of the market portfolio. Asset specific information accumulated over non-trading days is then aggregated onto the first day of trade following a non-trading period. They then estimate betas through an iterative GLS procedure.